

## **Developing Mathematical Ideas and Common Core State Standards Making Meaning for Operations**

By focusing on central mathematical ideas across the grades, the DMI modules support teachers in understanding how these ideas develop and what it looks like as students grapple with aspects of these ideas in a variety of problem contexts. It is not enough for teachers to know only the core work of their own grade levels. First, any classroom will include a range of students who are working at different places in their own understanding. Teachers need to recognize how students are building ideas that precede and follow those that are expected at their grade level. Second, teachers themselves should develop as deep and complete an understanding of these ideas as possible—both for their own learning and as a basis for making instructional judgments.

DMI was intentionally developed to support the kind of coherence and focus in the professional development of teachers of the elementary and middle grades to which the CCSS also aspires. Focus is provided by the selection, for each DMI module, of core mathematical ideas that underlie a key segment of mathematics content, while coherence comes from the careful analysis of how these core ideas connect to each other and are developed and applied by students across the grades. DMI is designed to help teachers understand these core ideas more deeply for themselves and gain extensive knowledge about how students engage with the progression of these ideas.

### **MMO and the CCSS Content and Practice Standards**

MMO, as its title indicates, focuses on the meaning and behaviors of the four arithmetic operations—addition, subtraction, multiplication, and division—with whole numbers and with fractions. In order to understand how students build an understanding of the operations, we need to step back from our own familiarity with them and reexamine their complexity, identify central aspects of each, and consider what is difficult for students. An operation, such as addition or subtraction, is not a set of instructions to compute a single result. Rather, each operation is an object of study, with its own set of properties and behaviors, its associated problem types, models, and applications, and its own relationships to other operations.

In this module, teachers think through the meaning of each of the four operations on whole numbers, how they are related to each other, what kinds of situations they model, how each can be represented, and how these meanings must be extended and deepened to accommodate fractions. The print and video cases provide examples of how students build this knowledge.

The material of this module is the substance of the CCSS domains, Operations and Algebraic Thinking (K-4) and Number and Operations—Fractions (3-5), which develop a progression of understanding the operations with whole numbers from kindergarten to grade 4, and expanding that understanding to include fractions, starting in grade 3.

MMO engages teachers in connecting the mathematical ideas of the CCSS with student learning. The Progressions for the CCSS-M on K Counting and Cardinality, K-5 Operations and Algebraic Thinking, and Number and Operations—Fractions 3-5 (<http://ime.math.arizona.edu/progressions/>) provide detailed top-down descriptions of the CCSS expectations for how this content builds. MMO immerses teachers in a bottom-up view of what this learning actually looks like as real students in real classrooms grapple with these important mathematical ideas, encountering each of the four arithmetic operations, first with whole numbers, then with fractions.

Two of the Standards for Mathematical Practice are emphasized in this module: MP2, Reason abstractly and quantitatively, and MP3, Construct viable arguments and critique the reasoning of others. MP2 is a focus of the work throughout the module. Reasoning abstractly and quantitatively has to do with decontextualizing problems expressed as situations and contextualizing problems expressed numerically. That is, a central part of understanding the meaning of the operations is being able to see how that meaning is expressed as relationships or actions on quantities in a context, how it is expressed with numbers and symbols, and to move fluidly between the two. As students describe the relationship between symbols and context, they are often also engaged in MP3—constructing and critiquing arguments about the quantities in a problem and the nature of the result.

In addition to the Mathematical Practices described above, MP1, Make sense of problems and persevere in solving them, is central to the work of MMO and, in fact, to all of the DMI modules. A central goal of DMI is to develop, support, and extend the mathematical thinking and reasoning of teachers as adult learners so that they can better support the mathematical thinking and reasoning of their students. As teachers make sense of mathematics for themselves and have the experience of persevering to solve problems they may at first think they cannot solve, they are also learning how to “develop a mathematics pedagogy in which student understanding takes center stage [MMO FG, p. 1].”

### **Materials needed to conduct a DMI Making Meaning for Operations seminar**

#### Published

Each participant in the seminar will need a copy of the Making Meaning for Operations (MMO) Casebook (© EDC 2015). <https://wwwcreatespace.com/4948545>

Each facilitator will need a copy of the MMO Facilitator’s Guide and a copy of the MMO DVD.

The main guide for the seminar is the Facilitator’s Guide. This includes the detailed agendas with handouts to support the math activities, casebook discussions, and homework assignments as well as suggestions for exit card prompts. The detailed agendas also provide guidance on timing and structures for the small and whole group discussions as well as a listing of the main points that should come up in each discussion and suggested questions for facilitators.

The Facilitator's Guide also includes Maxine's Journal. This document, written in first person in the voice of a seminar facilitator, offers a rich and detailed narrative describing a fictional MMO seminar. It offers facilitators a view of the kinds of math discussions and pedagogical issues that might arise. While it is fictitious, the events described in Maxine's Journal are based on seminars conducted during field-testing. Maxine's Journal illustrates the mathematical flow of the seminar as well as a glimpse at how teachers engage with seminar ideas and what they learn. While Maxine's Journal was written before the advent of the Common Core, the math and pedagogical discussions are compatible with both the content and practice standards.

#### Supplementary Materials available online

The supplementary materials provide additional information which make explicit the links between the DMI seminar and the CCSS. The supplement is used in concert with the published materials.

Session-by session notes include:

- A participant overview for each of the eight sessions. This overview should be distributed to participants at each session. It includes a listing of the agenda topics and timing for each component, a statement of the main mathematical themes for the session, a list of the Mathematical Practice Standards highlighted in the session, and a statement of the Common Core Math Content Standards addressed in the session.
- Notes to the facilitator regarding which seminar activities provide opportunities to reference one of the Math Practice Standards. These are offered as examples of opportunities where reference to one of the MPS can be made. Facilitators are likely to note additional examples and should call attention to them when they arise.
- A description of the modifications to be made to the published agenda including rewritten Math Activities, Focus Questions, and Homework Assignments for some sessions.