Session Three Overview

Making Meaning for Operations: Session 3

Agenda

Sharing Exit Card Comments	Whole group	5 minutes
Chapter 3 Case Discussion	Small groups Whole group	30 minutes 30 minutes
Math Activity: Comparing Fractions	Small group	35 minutes
Break		15 minutes
Math Activity (cont.)	Whole group	30 minutes
Viewing the DVD: Comparing Fractions	Whole group	30 minutes
Homework and Exit Cards	Whole group	5 minutes

Mathematical Themes

- As students extend their concept of number to include fractions and zero, their ideas about the behavior of the operations must be reconsidered.
- The same quantity can be represented by different fraction names depending on which is taken as 1, the unit or the whole.
- The value of a fraction is determined by the relationship between the numerator and the denominator.

<u>Connections to the Common Core: Standards for Mathematical Practice</u>

MP1 Make sense of problems and persevere in solving them.

MP3 Construct viable arguments and critique the reasoning of others.

MP5 Use appropriate tools strategically.

Connections to the Common Core: Content Standards

Grade 3: Number and Operations - Fractions
Grade 4: Number and Operations - Fractions
Grade 5: Number and Operations - Fractions

3. NF.1. *Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.*

3.NF. 2. *Understand a fraction as a number on the number line; represent fractions on a number line diagram.*

a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3. NF.3 *Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.*

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

4 .NF.1. Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4. NF. 2. *Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.*

5. NF. A.2 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Facilitator Note: An Example of integrating content and practice standards in MMO

The Standards for Mathematical Practice are intended to be integrated with the Mathematics Content Standards. Students can only learn how to engage in these practices meaningfully in the context of engaging with content. In this note, we provide an example of how content and practice standards are integrated in Session Three activities. An additional example is described in the Sessions Five materials. Facilitators should use these examples as guidelines for planning to integrate the content and practice standards in the other sessions as well.

In this session, participants begin their study of operations with fractions. They begin with a case discussion that focuses on first graders dividing 7 brownies equally among 4 people and upper elementary students considering the meaning of the problems 39 ÷ 5 and 5 ÷ 39. Division of whole numbers that do not divide evenly motivates the need for fractions as numbers. Particularly when the dividend is less than the divisor, students have to reconsider their understanding of the meaning and behavior of division, extending it to these new situations. Now that fractions have been introduced, participants think through the meaning of fractions *as numbers* in a math activity in which they compare pairs of fractions.

The work of this session introduces the definition of a unit fraction given in the CCSS: A unit fraction, $1/n$, represents the size of one part when dividing a whole into n equal parts. m/n represents m of those equal parts. In order to divide 7 brownies among 4 people, it is necessary to divide whole brownies into some number of equal parts; now we need a name for these quantities that are less than 1. The ideas in this session relate to the CCSS content standards that focus on understanding a fraction as a number or quantity (grades 2-3), using words and fraction notation to describe fractional quantities (grades 2-3), solving division word problems that result in a fraction or mixed number (grade 5), and comparing fractions (grades 3-4). In order to expand understanding of operations to include operations with fractions, students must first understand what these new numbers are, how fractional quantities relate to a whole, and how to interpret how fraction notation models those quantities.

Participants are also working with MP3, Construct viable arguments and critique the reasoning of others in this session, both as they discuss the cases and as they work on the math activity. In Cases 13 and 14, students are considering the meaning of 5 ÷ 39 and the nature of the quotient. Some students at first contend that "You can't divide a number that's lower by one that's higher." As the discussion continues and students put forth and critique various arguments, the teacher asks them to justify their arguments by referring to a context (5 candy bars divided equally among 39 people) and representations of that context. In the math activity, participants determine which of two fractions is greater and attempt to develop general arguments for comparing certain categories of fractions. For example, when comparing 4/5 and 4/7, participants might argue that because fifths are greater than sevenths, four of the fifths must be greater than four of the sevenths. In responding to the question, “What general statements about the relative sizes of fractions can you make based on these examples?” participants might argue for any two fractions with the same numerator, the greater of the two is the fraction with the smaller denominator.

The cases for this session also provide good examples of MP1, Make sense of problems and persevere in solving them. In Cases 13 and 14, students are making sense of and comparing the division expressions 39 ÷ 5 and 5 ÷ 39. These cases also provide an interesting example of persistence, not just across several class sessions, but from one ‘[year to the next.

Notes to the Facilitator Regarding the Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them. *Mathematically proficient students at the elementary grades explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway.*

Cases 13 and 14 provide examples of students persevering as they work through their confusions regarding dividing a smaller number by a larger one. In the discussion, they begin by considering situations that require whole number answers, then revise their contexts to include situations which can include breaking objects into portions less than one. The students maintain interest in this discussion because their previous ideas are challenged and they continue to try to make sense of the problem. As participants discuss these cases, point out the links to MP1.

MP3 Construct viable arguments and critique the reasoning of others. *Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions…. Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. They can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.*

In case 14, Mitchell comments, “So, if each kid was going to get equal shares, they would have to cut the candy bar into little equal pieces,” which leads the class into new ways of thinking about the problem. In the case discussion, help participants analyze the contributions of various students. Point out how the students are working to follow, question, and refine each other’s reasoning. Ask how they see MP3 being illustrated in this case.

Case 16, in which students discuss dividing by 0 and dividing into 0, provides additional examples of students engaged in MP3. For example, in the conversations started by Hayley, Jacob, and Cameron, the students question each other's reasoning and explain their own ideas as they discuss the role 0 plays in the number system.

MP5 Use appropriate tools strategically. *Mathematically proficient students at the elementary grades consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. These tools might include physical objects (cubes, geometric shapes, place value manipulatives, fraction bars, etc.), drawings or diagrams (number lines, tally marks, tape diagrams), paper and pencil, rulers and other measuring tools, scissors, tracing paper, grid paper, arrays, virtual manipulatives or other available technologies. Proficient students are sufficiently familiar with tools appropriate for their grade and areas of content to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from their use as well as their limitations.*

Mathematical tools for reasoning include the number line and fractional relationships such as fraction landmarks. During the math activity, Comparing Fractions, participants will use the number line as a tool for ordering fractions. They will also use landmark fractions to compare fractions, as in " I know 3/8 is less than 4/5 because 3/8 is less than 4/8, 4/8 is the same as ½ and 4/5 is larger than ½." Highlight these examples of MP5 when they occur.

Facilitator Note: *CCSS and the definition of fractions*

In Session Three participants begin their work with fractions. CCSS relies on a specific definition, building on the concept of a unit fraction ($1/n$), the quantity formed by one part when dividing a whole into n equal parts. A non-unit fraction (m/n) is the quantity formed by m of those unit parts ($1/n$). For instance, 1/3 is defined as the size of one part when dividing a unit into three equal parts. 2/3 is the size of 2 parts each equal to 1/3.

Facilitator Note: *Apply and extend understanding of whole numbers to fractions.*

Throughout the rest of the module participants engage with an overarching idea in CCSS: As the number system is extended from whole numbers to fractions, ideas that were consolidated for whole numbers are revisited to see what stays the same and what needs to be modified to incorporate these new kinds of numbers.

For example, consider cases 13 and 14. When trying to make sense of 5 ÷ 39, the class initially comes up with only whole number contexts and does not think about numbers between 0 and 1. Months later students reconsider 5 ÷ 39, now creating a problem context that helps them think about a unit divided into parts.

MMO Session Three Agenda Changes linked to Common Core

There are 7 modifications to the agenda in Session Three.

1. Use a few minutes at the beginning of the session to share exit card comments. Be sure to include comments participants wrote about the mathematical practice standards.

2. Distribute the session overview.

3. A reflection has been added to the Focus Questions to support discussion of MP2. The Focus Questions have also been rewritten to align line numbers in the casebooks with the questions. See below for a copy of the revised Focus Questions: Chapter 3.

4. Use the last five minutes of the whole group discussion of the Math Activity: Comparing Fractions to explain the definition of fraction used in CCSS as described in the facilitator note, *CCSS and the definition of fractions*.

5. Delete the activity, Planning for the Student-Thinking Assignment and replace it with the following:

   Viewing the DVD: Comparing Fractions (30 minutes)
   Whole group

   The DVD clip builds on the previous discussion regarding comparing fractions. Two students (Darrell and Johnny) are playing a game that involves placing fractions on a number line.

   In order to help participants attend to the mathematics in the clip, first ask them to think briefly about ordering the following three fractions: 3/8, ¼, and 1/3.

   Tell participants that on the first viewing, they should concentrate on following the mathematical arguments the students offer. Let them know you will show the video a second time to focus on the teacher's questions. On the second viewing post this question to guide the discussion, "What was the role of the teacher and each of the students in constructing and critiquing an argument?"

   Note: Alert the group to the fact that the students use the phrase ½ out of 3 to mean ½ divided by 3 in the same way that 2 out of 3 means 2/3. Also, at one point in the clip Johnny says, "I'm blocked." He is referring to the game they are playing and doesn't mean he is blocked in any other way.

   The sound on the DVD is difficult to hear at times. A transcript of this video clip is provided below.

6. The Fourth Homework sheet has been revised. One of the diagrams in Chapter 4 of the casebook was published incorrectly. A correct version has been added to the homework sheet. See below for a revised copy of the Fourth Homework.n

7. Exit Questions for Session Three:

   What mathematics are you wondering about from this session?

   What comments or questions regarding MP1 or MP3 did this session bring up?

DVD Transcript: Darrell & Johnny
Fifth Grade

Darrell: I choose, I mean I chose 1/3 here because I know that it's, um, it's more than ¼ because the denominator is smaller and since the denominator is smaller the pieces are going to be bigger. And it's less than ½. And since 2/3 is already more than 1/2, 1/3 is going to be less.

Teacher: How much less?

Darrell: Um, 1/3.

Teacher: How much less than ½ is 1/3?

Johnny: ½ of 1/3.

Teacher: OK.

Johnny: I'm blocked.

Teacher: You're blocked? I have one last question for you guys. I'm just going to take your attention to 3/8, ok, 3/8 and these two cards. Let's ignore 1/3 for now, ok? Which one is greater [3/8 or ¼] and how do you know?

Darrell: This one [3/8] is greater because 2/8 is equivalent to ¼, but this one has one more eighth.

Johnny: I would say 3/8 is um larger, because ¼, um 2/8 is equivalent to ¼. And 3/8 and, and 2/8 has, and 3/8 is 1/8 higher than the ¼, [or equivalent to.]

Teacher: OK. And now how about this one? How do we know that 3/8 is bigger than 1/3?

Darrell: Because. Um, 3/8, um. 8 is bigger than the… The denominator is bigger than the… never mind. Oh wait, the denominator is bigger so the pieces will be smaller but, and 1/3 you only need three pieces so the…

Teacher: What do you think, Johnny?

Johnny: I think that 3/8 is bigger, since, like. 8th is a larger, is like the larger denominator, so it needs smaller pieces, but I would say that 3/8 because if I want to get to a half, then I would be like, um add like, 1 and ½ out of 3 and ½ out of 3 would be like less than 1/3, is like, would be ha-- , larger than 1/8.

Teacher: Larger than 1/8. Can you repeat what you just said? I kinda got lost in the words.

Johnny: Um, I would say that 3/8 is larger because, um, 3, because 3/8 means needs 1/8 to get to a whole…

Teacher: …1/8 to get to…a whole?

Johnny: …a half.

Teacher: A half. OK, I'm just going write while you're talking to me so I understand it. Is that ok?

Teacher: So zero, one whole, and one half. And you just said, what

Johnny: 3/8 needs 1/8 more to get to a half.

Teacher: 1/8 more to get to a half. OK. Here's my 3/8, right? And you said it takes one more 1/8 to get to a half? And what about [3/8] 1/3?

Johnny: And 1/3 if I want to get to ½, I would, I would add, um, a half out of 3. A half out of 3 is larger than 1/8.

Teacher: So you said ½ out of 3 is larger than 1/8. Like that? OK.

Johnny: And so 3/8 would be larger.

Teacher: Is there anything equivalent to ½ out of 3? That's a weird fraction.

Darrell: Um.

Teacher: What would be ½ out of 1/3?

Darrell: 1 out of 6.

Teacher: Hmm?

Darrell: 1 out of 6.

Teacher: 1/6? How do you know that?

Darrell: Because I doubled the ½ to get 1, and I doubled 3 to get 6…

Teacher: OK. How about you?

Johnny: And 1/3 is, 1/3 is equivalent to 2/6, so ½ of 3 would be equivalent to 1/6.

Teacher: OK, so I'm going to erase this and write 1/6. Does that sound good? And you said "1/6 is further away from the ½, than 1/8" Is that true?

Darrell: Yeah. You would need a bigger jump.

Teacher: Bigger jump. Alright.

SESSION 3

Focus Questions: Chapter 3

1. In MaryAnn's case 13, T.C. declares, "You can't divide a number that's lower by one that's higher." Why does this statement seem true? In what ways is it not true? Why does MaryAnn think the idea of "fair shares" would help T.C. and his classmates look at division differently?

2. In case 14, Darrell says, "I think 39 can't go into 5. I mean, it can go into it, but it's going to be a fraction, it's got to be a fraction. A larger number into a smaller number—5 can go into 39, but there's a remainder. No, it's not a remainder; it's not a number." What might Darrell be thinking? What do you think he means by "number"?

3. The students in MaryAnn's two cases realize that, in order to think through the difference between 5 ÷ 39 and 39 ÷ 5, they could use story contexts that are modeled by the arithmetic expressions. Modify their stories or make up some story contexts of your own that help you think about this.

4. Trace the development of the discussion in case 14 from Mitchell's comment at line 185 to the end of the case. Explain what math ideas the following students add to the conversation: Leo, Cynthia, Tori, Maribel, Laila, Anthony, and Alejandro.

Reflection: Look over your work for questions # 1-4. How is the work of these students related to MP3, Construct viable arguments and critique the reasoning of others? Identify specific passages that illustrate this mathematical practice.

5. Explain the diagram offered by Cynthia and Leo near the end of case 14 (line 239) and consider the questions posed by the class at the end of the case (lines 244–259).

6. In Jayson's case 16, his friend Charlene's middle school students, who are exploring what it means to divide into 0 and to divide by 0, surface both important relationships and misunderstandings about the operations of multiplication and division and the nature of 0. Examine the following passages to discuss this: Cameron beginning at line 417 (including his drawings), Cameron beginning at line 493, Ashley beginning at line 497, and Renee at line 502.

Reflection: Look over your work for question #6. How is the work of these students related to MP3, Construct viable arguments and critique the reasoning of others? Identify specific passages that illustrate this mathematical practice.

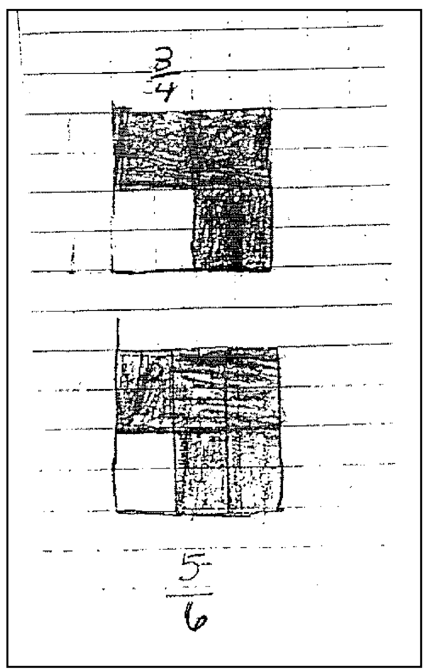
SESSION 3

Fourth Homework **Page 1**

Reading assignment: Casebook chapter 4

In the casebook, read chapter 4, “Greater Than, Less Than, Equal To,” including the introductory text and cases 17 to 21. Use the questions posed in the introduction as a guide for your reading of the cases.

One the diagrams in the 2010 edition of the casebook was printed incorrectly. If you are using that version, replace Figure 4.1 of Case 18 with this drawing.



Writing assignment: Sharing brownies

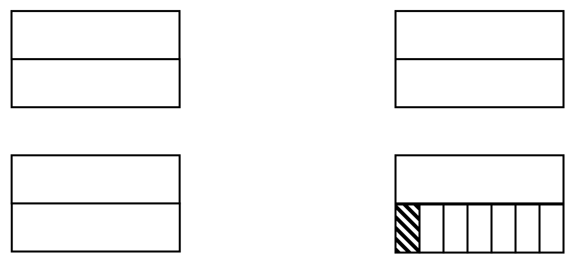
Ask your students to work on the brownie problem 4 children sharing 7 brownies.

For kindergarten and grade 1 classes, you can modify the original problem to 4 children sharing 5 brownies or 4 children sharing 6 brownies.

For middle school classes, you might also pose the following variation:

Variation: I have 4 brownies and I want to share them among my 7 friends. How many brownies would each friend get? Solve this problem with a diagram and with an arithmetic sentence. Describe the connections you see between the diagram and the arithmetic.

One student offered this solution to the problem:



Each person gets 1/2 plus the shaded amount.

Is this correct? If so, explain why, and tell how it matches the answer you have.
If not, explain why it is incorrect.

Take notes on what your students did and the questions you asked to uncover their thinking. Select the work of four or five students to share at our next seminar meeting.

Write about what happened and what you learned by working with students on this task. Your writing should include the examples of student work, why each one is of interest to you, and what connections you note between their thinking and the MPS.

Please bring three copies of this writing to the next session.