Session Six Overview

BST Session Six

Agenda

Sharing Exit Card Comments	Whole group	5 minutes
Math Activity: Working with Decimals	Small groups	20 minutes
	Whole group	20 minutes
Chapter 6 Case Discussion	Small groups	25 minutes
	Whole group	15 minutes
Break		10 minutes
Math Game: "Under 1"	Whole group	10 minutes
	Pairs	15 minutes
	Whole group	10 minutes
Rules for Ordering	Small groups	20 minutes
	Partner and Whole group	15 minutes
Homework and Exit Cards	Whole group	5 minutes

Mathematical Themes

- Numbers between 0 and 1 can be expressed using an extension of the whole number place value system.
- There are different ways to represent decimal number with diagrams and physical objects; each must include a means to show that adjacent places have a ratio of 1 to 10.
- The same principles that govern whole number addition and subtraction apply to addition and subtraction of numbers involving decimals.
- Analyzing strategies for computation involving decimals provides a context for highlighting these principles.

Connections to the Common Core: Standards for Mathematical Practice

MP5 Use appropriate tools strategically.

MP6 Attend to precision.

MP7 Look for and make use of structure.

Connections to the Common Core: Content Standards

Grade 1: Number and Operations in Base Ten 3
Grade 4: Number and Operations in Base Ten 2
Grade 4: Number and Operations - Fractions 2, 6, and 7
Grade 5: Number and Operations in Base Ten 3 and 7

1.NBT.3: Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

4.NBT.2: Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

4.NF.6: Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

4.NF.7: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

5.NBT.3: Read, write, and compare decimals to thousandths.

a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000).

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

6.NS.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

An Example of Integrating Content and Practice Standards in BST

The Standards for Mathematical Practice are intended to be integrated with the Mathematics Content Standards. Students can learn how to engage in these practices only while engaging with content. This facilitator note illustrates how the content and practice standards are integrated in Session Six. Sessions Two and Three contain a similar note. Facilitators should use these examples as a guideline for planning to integrate content and practice standards in Sessions Seven and Eight as well.

A major theme of the CCSS is to apply and extend previous understandings of whole numbers as students work in other domains. In this session, participants extend their work on the structure of the base ten system to consider numbers less than 1 represented as decimals. In the math activity, they represent decimal numbers using graph paper, base ten blocks, cubes, or other materials, and then use those representations to add and subtract with decimals. In the case discussion, they consider how students grapple with extending the place value system, making sense of decimal notation, representing decimal quantities, and realizing how their knowledge of addition of whole numbers relates to addition with decimals.

Content in this session relates to the CCSS content standards across the grades that focus on understanding the meaning of decimal quantities and decimal notation and adding with decimals. It includes, for example, the grade 4 standard that students compare two decimals to hundredths that refer to the same whole by reasoning about their size (4.NF.7), the grade 5 standard that students read, write, and compare decimals to thousandths (5.NBT.3), and the grade 5 standard that students add decimals to hundredths, using concrete models or drawings and strategies based on place value, and properties of operations (5.NBT.7).

This session also continues the work on MP7, Look for and make use of structure. In this session participants consider both their own understanding of how to extend the structure of the base ten number system to places to the right of the decimal point and how students come to understand and operate with these numbers. As they create representations during the math activity to solve problems such as 0.68 + 0.045, they think through what the digits in each of these numbers represents, and how the structure of the base ten system remains consistent; that is, each digit is ten times smaller than the same digit in the place to its left (0.6 x 10 = 6). Participants also consider how the structure of addition strategies that depend on place value remains consistent when adding decimal numbers. In Case 27, students are solving a problem that involves adding 1.14 grams, .089 grams, and .3 grams of gold. One group gets the solution of 2.06 when they line up the numbers vertically and align them to the right, as if they were whole numbers. Another group lines up the numbers by place and gets the correct solution, 1.529 grams. The fifth grade class discussion focuses on the structure of the base ten system and why their strategy has to take into account the place value of each digit in their addends. When analyzed conceptually, the method for adding decimals is the same as that for adding whole

numbers—both involve adding the digits in like places.

Students in this case also engage in MP1, Make sense of problems and persevere in solving them. In this case, the students make sense of what quantities each of the numbers represent and, therefore, how to combine them. Although many of the students are stuck at various points in the discussion, both the students and teacher persist. When a group of boys represents the quantities with base ten blocks, many of the students relate to this image, understanding why tenths have to be combined with tenths and hundredths with hundredths.

Major Focus: *Apply and Extend Understanding of Whole Numbers to Decimals.*

In addition to the specific standards listed below, Session Six emphasizes an overarching idea in the Common Core: As the number system is extended, it is necessary to revisit ideas about whole numbers to see how they are extended or revised to accommodate new kinds of numbers. In this session, participants reexamine rules for ordering, adding, and subtracting whole numbers to consider what is the same and what changes for decimals.

Notes to the Facilitator regarding the Standards for Mathematical Practice

MP5 Use appropriate tools strategically. *Mathematically proficient students at the elementary grades consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. These tools might include physical objects (cubes, geometric shapes, place value manipulatives, fraction bars, etc.), drawings or diagrams (number lines, tally marks, tape diagrams), paper and pencil, rulers and other measuring tools, scissors, tracing paper, grid paper, arrays, virtual manipulatives or other available technologies. Proficient students are sufficiently familiar with tools appropriate for their grade and areas of content to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from their use as well as their limitations.*

In case 27, students figure out how to use base ten blocks to represent decimals and determine the sum of 1.14, 0.089, and 0.3. The representation helps these students explain the basic principle underlying addition of decimals, illuminating why the answer of 2.06 is incorrect. In the seminar, participants will experience the standard as they develop approaches to representing decimals. Refer to Maxine's Journal line numbers 62 to 116 for an example of this kind of discussion. While Maxine does not explicitly cite MP5, you should make the connection explicit.

MP6 Attend to precision. *Mathematically proficient students at the elementary grades communicate precisely to others. They start by using everyday language to express their mathematical ideas, realizing that they need to select words with clarity and specificity rather than saying, for example, "it works" without explaining what "it" means. As they encounter the ambiguity of everyday terms, they come to appreciate, understand, and use mathematical vocabulary. Once young students become familiar with a mathematical idea or object, they are ready to learn more precise mathematical terms to describe it. In using representations, such as pictures, tables, graphs, or diagrams, they use appropriate labels to communicate the meaning of their representation.*

In the math activity, Rules for Ordering, participants articulate rules for ordering whole numbers and decimals. As they work to find language to express their ideas precisely, participants enact MP6.

MP7 Look for and make use of structure. M*athematically proficient students at the elementary grades use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference), and attributes of shapes to solve problems*

In Session Six, participants revisit the models they made in Session Two for representing the base ten structure of number to see how to extend or modify those representations to include decimal numbers. As they work on the math activities, asking questions such as, "How does this model show the one-to-ten ratio between adjacent places?" or "What mathematical structures are incorporated in this representation?" can point out the links between their work and MP7.

NCTM's Mathematics Teaching Practices

While the focus on eliciting and following student thinking continues through all sessions of BST, discussion of teacher moves and teaching practice is also a component of a DMI seminar. Before the end of this session, take a few minutes to distribute the list of Mathematics Teaching Practices, from the National Council of Teachers of Mathematics 2014 publication *Principles to Actions*, which is copied below.

BST Session Six Agenda Changes linked to Common Core

Note: If your seminar is comprised primarily of teachers of grades K – 4, you may decide to extend time for the math work on representing, ordering, and adding and subtracting decimals across both Session Six and Session Seven. This requires discarding one or more of the discussions on multiplying and dividing decimals in Session Seven.

There are six modifications to the agenda in Session Six.

1. Use a few minutes at the beginning of the session to share exit card comments.

2. Distribute the session overview.

3. Replace the activity Examining Curriculum Activities with "Rules for Ordering." See below for the handout, Rules for Ordering.

4. Revised exit card questions for Session Six:

    - What connections between decimals and whole number place value were highlighted for you in this session?
    - Consider one of the Standards for Mathematical Practice highlighted at today's session (#5, #6, and #7) and explain how it was enacted in your experience.

5. Distribute the handout, Mathematics Teaching Practices. Explain that this document is from the 2014 publication of the National Council of Teachers of Mathematics, *Principles to Actions*,[1] and that it lists teacher actions designed to support their students in developing the Standards of Mathematical Practice of the Common Core. Let the group know they should refer to this as they complete the chapter 7 reading assignment.

6. The student thinking assignment has been rewritten to include references to the Standards for Mathematical Practice. See below for the rewritten assignment.

[1] National Council of Teachers of Mathematics (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA.

Math Activity: Rules for Ordering

In lines 348 to 358 of case 26, the teacher, Nicole, has posted rules the class uses when comparing two whole numbers.

1. *The number that has more digits is larger.*
2. *If both numbers have the same number of digits or columns, start comparing from the left. The first one that has a higher digit is higher.*

Now Nicole asks, "Would the rules for decimals be the same as the rules we made for whole numbers?" As the case continues, some students make progress, but rules for comparing decimals are not stated and the question is left unanswered.

In this activity, take on this question for yourselves. Write out a rule for comparing two decimal numbers.

Does your rule for decimals also work for comparing two whole numbers? If not, revise your rule so that it applies to both whole numbers *and* decimals.

SESSION 6

Seventh Homework

Reading assignment: Casebook chapter 7

In the casebook, read chapter 7, "Multiplying and Dividing with Decimals," including the introductory text and cases 28–30. Use the questions posed in the introduction to guide your reading.

The teachers in these cases make a number of strategic moves to draw their students' attention to particular mathematical issues. Some of these moves are questions they pose to the class; others are seen in classroom activities they set up. As you read the cases, take note of specific decisions or actions of the teachers. Use the list of Mathematical Teaching Practices to categorize the teacher moves. If you have identified teacher moves that do not appear on this list, describe them in your own categories. Mark each move with a sticky note or make use of relevant line numbers so you can easily locate them at the next session.

Writing assignment: Writing about student thinking

Select a mathematics task to your students related to the work of this seminar. You might pose a question taken directly from one of the cases or the mathematics work you have done. In addition to planning for this math content, you should also choose one or two of the Standards for Mathematical Practice to enact during this class session.

After the session, think about what happened. What did you expect? Were you surprised? What did you learn? Your reflection should also include which of the mathematical practices you chose, how you approached it, and its impact on the students.

Write up your question, how your students responded, and what you make of their responses (your expectations, your surprises, and what you learned). Include specific examples of student work or dialogue. Reporting in detail about the work of a few students is very helpful. In particular, it is useful to analyze the thinking of students whose work might be confusing.

At our next session, you will have the opportunity to share this writing with colleagues in the seminar. Please bring three copies of your writing to share and to turn in.

NCTM Mathematics Teaching Practices

Establish mathematics goals to focus learning.

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving.

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations.

Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse.

Effective teaching of mathematics facilitates discourse among students to build share understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions.

Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding.

Effective teaching of mathematics builds -fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics.

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking.

Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.