Session Three Overview

BST Session Three

Agenda

Sharing Exit Card Comments	Whole group	5 minutes
Sharing Math Interviews	Pairs	20 minutes
Math activity: Addition and Subtraction Strategies	Whole group Small groups	15 minutes 30 minutes
Break and Viewing Posters		20 minutes
Discussion: Addition and Subtraction Strategies	Whole group	25 minutes
DVD: Session Three	Whole group	20 minutes
Case discussion: Chapter 3	Small groups Whole group	20 minutes 20 minutes
Homework and Exit Cards	Whole group	5 minutes

Mathematical Themes

- Computational strategies for multi-digit addition and subtraction rely on the place value system and properties of the operations.
- Examining the reasoning that underlies addition and subtraction procedures, including the standard algorithms, provides a context for students to deepen their understanding of the place value system and of the operations themselves.

Connections to the Common Core: Standards for Mathematical Practice

MP7 Look for and make use of structure.

MP8 Look for and express regularity in repeated reasoning.

Connections to the Common Core: Content Standards

Grade 1: Numbers and Operations in Base Ten 4 & 6
Grade 2: Numbers and Operations in Base Ten 5, 6, 7, 8 & 9
Grade 4: Number and Operations in Base Ten 4

1.NBT.4: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method

and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

1.NBT.6: Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

2.NBT.5: Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6: Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.7: Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.8: Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.NBT.9: Explain why addition and subtraction strategies work, using place value and the properties of operations.

3.NBT.2: Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Facilitator Note: An Example of Integrating Content and Practice Standards in BST

The Standards for Mathematical Practice are intended to be integrated with the Mathematics Content Standards. Students can only learn how to engage in these practices meaningfully in the context of engaging with content. In this note, we provide an example of how content and practice standards are integrated in Session Three seminar activities. Examples based on Sessions Two and Six are described in those sessions. Facilitators should use these examples as guidelines for planning to integrate the content and practice standards in the other sessions as well.

In this session, participants apply their knowledge of place value to make sense of addition and subtraction strategies and algorithms. In the math activity, they represent and describe common addition and subtraction strategies and algorithms, explain how each works, and identify the underlying mathematical ideas. In doing this work, teachers focus on the meanings of each operation and of each number in an addition or subtraction expression, properties and behaviors of the operation, decomposition of numbers by place, and other uses of the place value system to solve problems (e.g., adding on an amount to create a multiple of 10 in order to make a problem easier to solve). In analyzing the standard addition and subtraction algorithms, teachers make sense of what the "carried" or "borrowed" number means in the base ten system in order to understand why these procedures work. In the cases, teachers read about students who analyze and compare strategies and algorithms as their teachers help them make sense of the operations and the base ten structure of the numbers.

The work of this session relates to the CCSS content standards across the grades that focus on developing fluency with addition and subtraction. It includes, for example, the grade 2 standard that students fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (2.NBT.5) and the grade 4 standard that students fluently add and subtract multi-digit whole numbers using the standard algorithm (4.NBT.4).

MP7, Look for and make use of structure, is prominent in this session. In the math activity, participants describe and analyze a variety of important structures, including the base ten system and properties of the operations. For example, in describing strategy (1a) in the math activity, in which 34 + 28 is broken up by place, teachers need to pay attention to the meaning of each digit in the base ten system—that the 3 represents 3 tens and the 2 represents 2 tens, which can then be combined to get 5 tens. They also need to pay attention to the structure of the operation of addition, which accounts for why, once 34 and 28 numbers are broken into parts, the parts can then be arranged in any order and recombined to determine the sum. The cases offer examples of how teachers help students to understand and apply these structures. In particular, the structures of addition do not apply to other operations. For example, in Case 13, a student solves 37 – 19 (37 pigeons, 19

flew away) by first solving 30 – 19 to get 11, but then is not sure how to "put back" the 7. When the teacher asks, "Did those pigeons leave or stay?" she uses the story context to focus the student on the structure of subtraction and the meaning of each quantity in a subtraction expression. Similarly, in Case 14 the teacher draws her students' attention to both the structure of subtraction and the structure of the number system when she asks, "How would you take this problem apart to make it easier to solve?"

The cases for this session also provide good examples of MP1, Make sense of problems and persevere in solving them. In all of the cases, students grapple with analyzing and justifying strategies, working together to clarify how and why those strategies work.

Notes to the Facilitator regarding the Standards for Mathematical Practice

MP7 Look for and make use of structure. M*athematically proficient students at the elementary grades use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference), and attributes of shapes to solve problems*

During the math activity and the case discussion, ask questions such as, "How does this strategy use the base ten structure?" or "When moving from considering addition to considering subtraction strategies, which structures still apply and which change?"

MP8 Look for and express regularity in repeated reasoning. *Mathematically proficient students at the elementary grades look for regularities as they solve multiple related problems, then identify and describe these regularities.*

In the math activity, after applying a strategy to a set of examples, participants are asked to write a verbal description of the strategy in general terms. At the end of the math discussion, talk about this process as an illustration of MP #8. Refer to Maxine's Journal line numbers 270 to 325 for an example of this kind of discussion. While Maxine does not explicitly cite MP8, you should make the connection explicit.

Note regarding MP7 and MP8

MP8 might be considered a category of MP7. In enacting MP8, participants notice something that recurs in specific instances and express that "regularity" in general terms. For instance, when participants create a verbal description of a procedure for doing addition—you decompose the numbers by place, add like places, and add the results—they are expressing regularity in the reasoning they employed in specific cases (MP8). When they call upon place value or the structure of addition to explain why the procedure will always work, they are engaged in MP7.

Note: The Common Core and the US Traditional Addition and Subtraction algorithms

In Session Three, building on the work of previous sessions, participants analyze a variety of computation algorithms including those traditionally taught in the United States. The Common Core approach to algorithms is similar: Formal work with the algorithms comes years after students explore computation with other methods. In grades 1, 2, and 3, students become fluent in a variety of strategies, and by the end of grade 4 they are expected to know the standard algorithms for addition and subtraction. In grade 5, they learn the standard multiplication algorithm; in grade 6, the standard division algorithm.

Case 14 illustrates how a fourth-grade teacher helps students move toward reasoning that underlies the standard subtraction algorithm.

Note: Additive and Multiplicative Structures

The use of the term structure in MP7 is somewhat opaque. The place value system of our numbers is an example of how the term structure can be applied to mathematics. But what does it mean to talk of the structures of addition and multiplication?

Consider this example. In this session participants will explore this strategy for adding 34 + 28: Decompose 34 into 30 + 4 and 28 into 20 + 8. Then add 30 + 20 and 8 + 4 arriving at 50 + 12 which equals 62. In words, this procedure can be expressed as; Add the tens, add the ones, the sum is the answer to the original expression. Properties of the addition such as commutativity and associativity are part of the structure of addition that supports this procedure.

In the next session, participants will examine procedures for multiplication. In so doing they will come to see that the structure of multiplication is different from that of addition. Trying to calculate 34 x 28 by applying a strategy similar to that used above—multiply the tens, multiply the ones, combine the answers—will not produce a correct result. The structure of multiplication includes the distributive property.

BST Session Three Agenda Changes linked to Common Core

There are six adjustments to the agenda in Session Three.

1. Use a few minutes at the beginning of the session to share exit card comments. Be sure to include comments participants wrote about the standards for mathematical practice.

2. Distribute the overview for this session.

3. Near the end of the math discussion: Addition and Subtraction Strategies, invite participants to articulate how their work in the activity illustrates MP8. Finally, ask, "What are times in your work with students when MP8 is enacted?"

4. In discussing the bulleted points for whole-group discussion: Addition and Subtraction Algorithms, point out the connections to MP7 when they occur.

5. Revised exit card questions for Session Three:

    - What mathematics are you wondering about from this session?

    - How did MP8 come up in the math work you did in this session? If you have questions about MP8, what are they?

6. The homework page has been rewritten to include a new reflection (question #3) on the standards for mathematical practice. See below for a copy of the new homework page.

SESSION 3

Fourth Homework

Reading assignment: Casebook chapter 4

In the casebook, read chapter 4, "Multiplication of Multidigit Numbers," including the introductory text and cases 15–18. Consider the questions posed in the chapter introduction as you read. Since the cases span grade levels 3 to 9, you may encounter ideas that are not typically covered in the grades you teach. The next seminar session will include opportunities for you to work on the mathematical ideas for yourself in both small-group and whole-group settings.

Writing assignment: Analyzing subtraction algorithms

1. Consider these two approaches for solving the subtraction problem 53 – 17.

(a) 53 – 17 = 56 – 20 = 36

(b)
$$\begin{array}{r} {}^{4}\cancel{5}\,{}^{1}3 \\ -\ 1\ \ 7 \\ \hline 3\ \ 6 \end{array}$$

2. Do the following to analyze each approach:

- Create a representation for the strategy using the kind of models explored in the session.
- Try another subtraction problem using this procedure.
- Explain how procedure works; that is, why does it produce a correct answer?
- Make a list of the mathematical structures involved in each procedure.

If you are unsure of your response to any of these, write out as much as you do understand, and explain where you get stuck and why.

3. Explain the links you see between your work on question 2 and one of the standards for mathematical practice.