Session Two Overview

BST Session Two

Agenda

Main idea of the Session Sharing Exit Card Comments	Whole group	10 minutes
Discussion: Norms for Learning	Whole group	20 minutes
Math activity: Multiplying by 10	Small groups Whole group	25 minutes 20 minutes
DVD for Session Two	Whole group	25 minutes
Break		15 minutes
Chapter 2 Case Discussion	Small groups Whole group	30 minutes 25 minutes
Homework and Exit Cards	Whole group	10 minutes

Mathematical Themes

- The value of a number is determined by multiplying the value of each digit by the value of the place that it occupies and then summing. For all whole numbers, the value of the place farthest to the right is 1; the value of all other places is 10 times the value of the place to its right.
- A study of the result of multiplying any number by 10 reveals aspects of the base ten structure of the number system.
- Examining the different ways in which number lines can be used to represent numbers highlights the relative magnitude of powers of ten.
- As students work to learn the number system, their errors highlight the mathematics that underlie the system; e.g., we say *one hundred ninety-five*, but we write 195, not 100905.

Connections to the Common Core: Standards for Mathematical Practice

MP3 Construct viable arguments and critique the reasoning of others.

MP7 Look for and make use of structure.

Connections to the Common Core: Content Standards

Kindergarten: Number and Operations in Base Ten
Grade 1: Number and Operations in Base Ten
Grade 2: Number and Operations in Base Ten
Grade 3: Number and Operations in Base Ten

Grade 4: Number and Operations in Base Ten
Grade 5: Number and Operations in Base Ten

K.NBT.1: Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 +8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

1.NBT. 2: Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a "ten."
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

2.NBT.1: Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."
b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

3.NBT.3: Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.

4.NBT.1: Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.

5.NBT.1: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

Facilitator Note: An Example of Integrating Content and Practice Standards in BST

The Standards for Mathematical Practice are intended to be integrated with the Mathematics Content Standards. Students can only learn how to engage in these practices meaningfully in the context of engaging with content. In this facilitator note, we provide an example of how content and practice standards are integrated in the Session Two seminar activities. Examples based on Sessions Three and Six are described in those sessions. Facilitators should use these examples as guidelines for planning to integrate the content and practice standards in the other sessions as well.

After considering how knowledge of place value is applied in addition and subtraction strategies in Session One, participants focus in Session Two on the structure of the base ten system. In the math activity, Multiplying by 10, participants multiply numbers by 10, 100, and other powers of ten. They create representations to help them describe what occurs and explain why it occurs. The cases of chapter 2 focus on how students learn to understand the place value system across the grades. These cases include students' attempts to coordinate and make sense of the way numbers are written and spoken, to represent small and large numbers on a number line, and to understand the structure of large numbers. Teachers consider common student errors, such as counting fifty-eight, fifty-nine, fifty-ten, and how such errors reveal students' logic as they are learning to make sense of the base ten system.

This work relates to CCSS content standards across the grades that focus on the following three aspects of our number system. One, the place value system is built on powers of ten. Two, the relationship between the places is multiplicative, a digit in any place represents ten times as much as the same digit in the place to its right. Three, written numbers and spoken numbers call upon this place value structure in different ways.

Content Standards relating to these ideas span the grades. For instance, a Grade 1 standard states that students understand that the two digits of a two-digit number represent amounts of tens and ones (1.NBT. 2) while a Grade 5 standard states that students recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left (5.NBT.1).

Participants are also working with MP8 (Look for and express regularity in repeated reasoning) in this session as they describe the regularity of the base ten system. For example, in problem 1 of the math activity, they consider the set of problems 23 x 1, 23 x 10, 23 x 100, 23 x 1000, and 23 x 10000. They consider the regularity of the progression of products (23, 230, 2300, . . .) and describe and explain why this regularity occurs. When they call upon the multiplicative structures in their representations they are enacting MP7. As they move from description to explanation, they are engaging in MP3 and MP6, constructing and articulating their arguments. Through this activity, teachers recognize these practices in their own learning, while the cases illustrate how teachers can help students learn to engage in these practices. For example, an instance of engaging students

in MP8 occurs in Case 9, in which second graders notice that when they divide a number of beans (e.g., 67) into groups of 10 and leftovers, the number of tens (e.g, 6) and the number of leftovers (e.g., 7) together create the numeral for the total amount; they notice that this only happens when they use groups of 10, not when they use groups of any other amount. The teacher has set up the activity and a way of recording that highlights this regularity for students. She then encourages them to express what they are noticing.

Notes to the Facilitator regarding the Standards for Mathematical Practice

MP3 Construct viable arguments and critique the reasoning of others. *Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions. … Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. They can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.*

During the discussion of group norms, include remarks about MP3. It is important that participants come to see that "critique" does not mean criticize. When they ask each other questions about their representations or solution strategies in order to understand them, they are critiquing each other's arguments.

At the end of the math activity (Multiplying by 10), ask participants to reflect on MP3 in relation to how they shared and followed one another's explanations

MP7 Look for and make use of structure. M*athematically proficient students at the elementary grades use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference), and attributes of shapes to solve problems)*

During the math activity and the case discussion, ask questions such as, "How does this representation show the base ten structure?" or "How does this students' thinking about how to write 'one hundred and ninety-five' with number symbols help you understand how he views the number system?" Refer to Maxine's Journal line numbers 200 to 240 for an example of this kind of discussion. While Maxine does not explicitly cite MP7, you should make the connection explicit.

Facilitator Note on MP4: Highlighting a Common Misconception

MP4 Model with mathematics. *When given a problem in a contextual situation, mathematically proficient students at the elementary grades can identify the mathematical elements of a situation and create a mathematical model that shows those mathematical elements and relationships among them.*

Note that the word, *model*, has two different definitions that apply in mathematics education. Frequently *model* indicates a physical representation, like Sarah's cubes in Lucy's Case 4 or the representations with base-ten blocks that participants use in the math activity as illustrated in lines 205 -215 of Maxine's Journal. These physical embodiments of mathematics are powerful tools to support the reasoning of participants and their students. However, they are not examples of MP4.

A second definition applies to MP4: a description of a situational context using mathematical concepts. In order to fit the meaning of MP4, the problem begins outside of mathematics in a real world context. Students abstract the mathematical aspects from the context, expressing those in mathematical language, diagrams, or physical representations. Examples of MP4 are highlighted in the DMI modules, *Patterns, Functions, and Change* and *Working with Data.*

In *Building a System of Tens* the word, *model,* refers to the first definition, a physical representation of a mathematical situation. This is unrelated to MP4. It might be necessary to clarify this distinction so that participants do not leave with the misconception that building models of numbers using base ten blocks is an example of MP4.

BST Session Two Agenda Changes linked to Common Core

There are six adjustments to the agenda in Session Two.

1. Use a few minutes at the beginning of the session to share exit card comments. Be sure to include comments participants wrote about MP7.

2. Distribute the overview for this session.

3. During the discussion regarding Norms for Learning, point out how the norms they generated are related to MP3. Clarify the difference between critiquing and criticizing.

4. During the math activity, ask questions such as, "How does your representation capture or illustrate the base ten structure of the number system?" After the discussion of question #4, when participants share their explanation of what happens when a number is multiplied by 10, invite participants to reflect on their experience and to note how MP3 was enacted in their discussion. If participants refer to MP4, clarify the distinction between modeling with mathematics and using blocks or materials to represent mathematical ideas as discussed in the facilitator note.

5. A new question (called Reflection) has been added to the Focus Questions. See below for a copy of the revised Focus Questions for Chapter Two

6. Revised exit card questions for Session Two:

    - How is the seminar working for you as a learner?
    - What ideas about the meaning of MP7 has the session illuminated for you? What questions about MP7 are you pondering?

SESSION 2

Focus Questions: Chapter 2

1. Turn to Dawn's case 6 and consider Andrew's suggestion. Why does it make sense to him to have "fifty-ten" follow 59? What is right in his thinking? What is he missing?

2. In Danielle's case 7, the students come up with many ways to write one hundred ninety-five. What sense do you see in each one?

3. What ideas about the number system does the activity in Donna's case 9 highlight?

Reflection: Look over your work for questions # 1-3. How is the work of these students related to MP7, Look for and make use of structure? What structures are the students calling upon?

4. Make a number line of your own from 0 to 10,000 and place 375 on it. Discuss how you decided where to mark the 375. Now place the following additional numbers on your number line:

   (a) 25 (b) 2,376 (c) 7,832

5. Compare Shaquille's and Chris's number lines in case 10. How are they alike? How are they different? What ideas about the number system do you see in these representations? What is not seen?

6. Explain Olivia's multiple number line strategy in case 10. What ideas about the number system do you see in her representations?

7. In Susie's case 11, she asks her students how many thousands there are in 437,812.

   (a) Trace the thinking of the students as they discuss this. What mathematical ideas about the place value system are coming up for them? What are they missing?

   (b) In line 550, the students use a calculator to determine that there are 437.812 thousands in 437,812. What does that mean?