Developing Mathematical Ideas and Common Core State Standards
Building a System of Tens

By focusing on central mathematical ideas across the grades, the DMI modules support teachers in understanding how these ideas develop and what it looks like as students grapple with aspects of these ideas in a variety of problem contexts. It is not enough for teachers to know only the core work of their own grade levels. First, any classroom will include a range of students who are working at different places in their own understanding. Teachers need to recognize how students are building ideas that precede and follow those that are expected at their grade level. Second, teachers themselves should develop as deep and complete an understanding of these ideas as possible—both for their own learning and as a basis for making instructional judgments.

DMI was intentionally developed to support the kind of coherence and focus in the professional development of teachers of the elementary and middle grades to which the CCSS also aspires. Focus is provided by the selection, for each DMI module, of core mathematical ideas that underlie a key segment of mathematics content, while coherence comes from the careful analysis of how these core ideas connect to each other and are developed and applied by students across the grades. DMI is designed to help teachers understand these core ideas more deeply for themselves and gain extensive knowledge about how students engage with the progression of these ideas.

BST and the CCSS Content and Practice Standards

BST, as its title indicates, focuses on the base ten number system—that system that underlies counting and computation with whole numbers and decimals. As adults, we so take this system for granted that we need to make a concerted effort to step back in order to study how students build understanding of this system, to analyze what its complexities are for new learners, and to consider how "mistakes" can signal important ideas. In this module, teachers think through the relationship between spoken numbers and written numerals, the meaning of each numeral in the system of place value, how computational strategies and algorithms are built on both the base ten structure of numbers and the properties of the operations, and how all of these ideas are extended as students move from working exclusively with whole numbers to working with decimals. The print and video cases provide examples of how students build this knowledge.

The material of this module is the substance of the CCSS domains, Number and Operations in Base Ten (K-5) and The Number System (grade 6), which develop a progression of the understanding and application of whole number place value from kindergarten through grade six, and a progression of the understanding and application of the place value of decimals in grades five and six. There are also related standards in grade 4 under Number and Operations—Fractions (understanding decimal notation and comparing decimal fractions).

BST engages teachers in connecting the mathematical ideas of the CCSS with student learning. The Progressions for the CCSS-M on K-5 Number and Operations in Base Ten and

The Number System, 6-8 (http://ime.math.arizona.edu/progressions) provide detailed top-down descriptions of the CCSS expectations for how this content builds. BST immerses teachers in a bottom-up view of what this learning actually looks like as real students in real classrooms grapple with these important mathematical ideas, extend and apply their understanding as they move from counting to computing with small numbers to multi-digit computation, encounter each of the four arithmetic operations, and extend their work with whole numbers to decimals.

Three of the Standards for Mathematical Practice are emphasized in this module: MP3, Construct viable arguments and critique the reasoning of others; MP7, Look for and make use of structure; and MP8, Look for and express regularity in repeated reasoning. MP7 is a focus of the work throughout the module. The cases illustrate how students come to understand the structure of the base ten system. They apply that understanding, along with structures of addition, subtraction, multiplication, and division, to develop computation strategies with whole numbers. Toward the end of the module, those structures are extended it to make sense of decimals as representations of quantity and to determine computation strategies with decimals As students develop, explain, and compare computation algorithms and strategies, they are engaged in MP8 and MP3. They identify and generalize strategies that work across problems and construct arguments to justify why such strategies do or don't work. Through critiquing faulty approaches, they come to more deeply understand the structure of the operations. MP6, Attend to precision, also comes into play: As students construct or critique arguments, working to make their reasoning clear to others, they are learning to pay attention to the precision of their language.

In addition to the mathematical practices described above, MP1, Make sense of problems and persevere in solving them, is central to the work of BST and, in fact, to all of the DMI modules. A central goal of DMI is to develop, support, and extend the mathematical thinking and reasoning of teachers as adult learners so that they can better support the mathematical thinking and reasoning of their students. As teachers make sense of mathematics for themselves and have the experience of persevering to solve problems they may at first think they cannot solve, they are also learning how to "develop a mathematics pedagogy in which student understanding takes center stage"

MP2, Reason abstracting and quantitatively, and MP5, Use appropriate tools strategically, are also at play in this module. They are mentioned at appropriate times in the sessions, but are not a main focus of *Building a System of Tens*.

Materials needed to conduct a DMI Building a System of Tens seminar

Published

Each participant in the seminar will need a copy of the Building a System of Tens (BST) Casebook (© EDC 2015). https://www.createspace.com/5249103.

Each facilitator will need a copy of the BST Facilitator's Guide and a copy of the BST DVD.

The main guide for the seminar is the Facilitator's Guide. This includes the detailed agendas with handouts to support the math activities, casebook discussions, and homework assignments as well as suggestions for exit card prompts. The detailed agendas also provide guidance on timing and structures for the small and whole group discussions as well as a listing of the main points that should come up in each discussion and suggested questions for facilitators.

The Facilitator's Guide also includes Maxine's Journal. This document, written in first person in the voice of a seminar facilitator, offers a rich and detailed narrative describing a fictional BST seminar. It offers facilitators a view of the kinds of math discussions and pedagogical issues that might arise. While it is fictitious, the events described in Maxine's Journal are based on seminars conducted during field-testing. Maxine's Journal illustrates the mathematical flow of the seminar as well as a glimpse at how teachers engage with seminar ideas and what they learn. While Maxine's Journal was written before the advent of the Common Core, the math and pedagogical discussions are compatible with both the content and practice standards.

<u>Supplementary Materials available online</u>

The supplementary materials provide additional information which make explicit the links between the DMI seminar and the CCSS. The supplement is used in concert with the published materials.

Session-by session notes include

- A participant overview for each of the eight sessions. This overview should be distributed to participants at each session. It includes a listing of the agenda topics and timing for each component, a statement of the main mathematical themes for the session, a list of the Mathematical Practice Standards highlighted in the session, and a statement of the Common Core Math Content Standards addressed in the session.

- Notes to the facilitator regarding which seminar activities provide opportunities to reference one of the Math Practice Standards. These are offered as examples of opportunities where reference to one of the MPS can be made. Facilitators are likely to note additional examples and should call attention to them when they arise.

- A description of the modifications to be made to the published agenda including rewritten Math Activities, Focus Questions, and Homework Assignments for some sessions.