

Session Seven Overview

Making Meaning for Operations: Session 7

Agenda

Sharing Exit Card Comments	Whole group	5 minutes
DVD for Session Seven	Individuals and pairs	10 minutes
	Whole group	25 minutes
Chapter 7 Case Discussion	Small groups	35 minutes
	Whole group	30 minutes
Break		15 minutes
Math Activity: Operating with Positive Rational Numbers	Small groups	35 minutes
Teacher Moves Discussion	Whole group	20 minutes
Homework and Exit Cards	Whole group	5 minutes

Mathematical Themes

- Problems involving multiplication and division by positive rational numbers require contexts that allow for the objects to be broken into parts.
- Diagram solutions for problems involving division of fractions can reveal the relationships among the operations; that is, a division of fractions problem can be solved by calling on addition, subtraction, or multiplication.

Connections to the Common Core: Standards for Mathematical Practice

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

Connections to the Common Core: Content Standards

Grade 5: Number and Operations - Fractions 7

Grade 6: The Number System -1

5. NF. 7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. 1 a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$. b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$ -cup servings are in 2 cups of raisins?

6. NS. 1. Apply and extend previous understandings of multiplication and division to divide fractions by fractions. 1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $3/4$ -cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?

Facilitator Note: *Apply and extend understanding of whole numbers to fractions.*

In Session Seven, participants continue to engage with an overarching idea in the Common Core: As the number system is extended from whole numbers to fractions, ideas that were consolidated for whole numbers are revisited to see what stays the same and what needs to be modified to incorporate these new kinds of numbers. In this session, participants will examine multiplication and division with fractions.

Notes to the Facilitator regarding the Standards for Mathematical Practice

MP2 Reason abstractly and quantitatively. *Mathematically proficient students at the elementary grades make sense of quantities and their relationships in problem situations. They can contextualize quantities and operations by using images or stories. They interpret symbols as having meaning, not just as directions to carry out a procedure. Even as they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects.*

In the DVD clip, Rachel generates story contexts for $4 \div \frac{3}{5}$ and says she could test her answer by multiplying. Ask participants to carry out the calculation and use the context to sort out the difference between $6 \frac{2}{5}$ and 6 and $\frac{2}{3}$. Call their attention to the process of reasoning by calling upon the context and then abstracting the mathematical elements from the context as they do this work. Offer this as an example of MP2.

The math activity, *Operating with Positive Rational Numbers*, offers participants opportunities to engage in MP2 as they generate story contexts for arithmetic expressions and work to make connections among their diagrams, the arithmetic expressions, and elements of the story context.

MP3 Construct viable arguments and critique the reasoning of others. *Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions.... Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. They can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.*

In case 29, as students work on the stretching elastic problem, they contrast and critique each other's diagram solutions and, in the process, generate new ways to approach the problem. Highlight this as an example of MP3.

MMO Session Seven Agenda Changes linked to Common Core

There are seven modifications to the agenda in Session Seven.

1. At the beginning of the session, select and share some of the exit card comments from the previous session.
2. Distribute the session overview.
3. After discussing the mathematics involved in the video clip (DVD for Session 7), ask how Rachel might have used the context to explain her error and make sense of the correct answer. Ask participants how this is related to MP2.
4. Math activity: Operating with positive rational numbers

When there are five minutes left in the Math Activity, let participants know that this math activity will be discussed in Session Eight. Suggest they use the next five minutes to review their work on this activity and write up their observations for the next session. If posters have been made, collect them for the next session.

5. Teacher Moves Discussion

Case discussion: Examining teacher moves

(20 minutes)

Pairs

Whole group

In this whole-group discussion, participants focus on the actions of the teachers in the cases and consider the impact of the teachers' actions on their students' thinking. Let the group know that as part of their homework, they will work with a case to analyze the teacher's moves. This brief whole-group discussion, based on case 29, is intended to provide an example of this kind of analysis.

Display the poster Discussion of Teacher Moves or distribute this as a handout (see below). Remind participants to gather their notes on the teacher moves in case 29 which they prepared for homework. Ask them to return to case 29 and discuss in pairs the classroom dialogue with a focus on what the teacher is doing. Allow five minutes of partner sharing before opening the whole group discussion.

6. Exit Questions for Session Seven:
 - How did this session work for you as a learner?
 - Explain something you figured out about the mathematical practices.
7. The writing assignment: Teacher Moves has been edited to include reference to the mathematical practices. See below for a new version of the Eighth Homework.

Discussion of Teacher Moves

- What is the teacher move?
 - What can you infer about the teacher's agenda for her students?
 - How does the teacher's action connect with where she thinks her students are?
 - What do you think the teacher is trying to accomplish through this move?
 - What is the impact of her move in terms of the ideas students engage with?
 - List examples of any of the Mathematical Practice Standards that are relevant in the teacher move or student work.
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Focus Questions: Chapter 7

1. In case 28, Sarita lists a variety of equations and explanations her students used to solve the ice-cream problem (see the boxed examples). Examine each of the statements. Which ones seem right, and why? Which ones seem wrong, and why? Your conclusions need not be the same as Sarita's.
2. In Selena's case 29, the students are working on a problem that involves the division of 33 by $5\frac{1}{2}$. Does this seem like a division problem to you? Why or why not? What ideas about division (or multiplication) were highlighted for you as you read the case? How are the ideas the same and how are they different from ideas involved with division (or multiplication) of whole numbers?
3. Examine the problems in this chapter in light of *Table 2, Common Multiplicative and Division Situations*. How would you categorize them?

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Eighth Homework

Reading assignment: Casebook chapter 8

In the casebook, read chapter 8, “Highlights of Related Research.” As you read, take notes on the following:

1. What did you learn from the research article that wasn’t in the cases?
2. Identify two points in the essay that particularly interested you. Explain what made them interesting.
3. What connections do you see between your classroom and the ideas in the essay?

Bring these notes to the next session to support discussion of this chapter.

Writing assignment: Teacher moves

For this assignment, you will write your analysis of the moves a teacher makes (in a case from the casebook) and the impact of that move on the students. Choose from these cases in *Making Meaning for Operations*:

Grades K – 2

Case 8 Bella or Case 12 Melinda

Grades 3 – 5

Case 6 Machiko or Case 17 Faith

Grades 6 – 8

Case 16 Jayson or Case 26 Sarita

1. Identify places in a case where the teacher’s moves interested you.
2. Analyze each passage by responding to these questions:
 - What did the teacher do?
 - What can you infer about the teacher’s agenda for her students?
 - How does the teacher’s action connect with her assessment of their understanding?
 - What do you think the teacher is trying to accomplish?
 - What is the impact of the move on the students’ thinking?
 - What links to the mathematical practices do you see?