Session Five Overview

Making Meaning for Operations: Session 5

Agenda

Sharing Exit Card Comments	Whole group	5 minutes
Chapter 5 Case Discussion	Small groups	30 minutes
	Whole group	30 minutes
DVD Joey's Run	Whole group	30 minutes
Break		15 minutes
Math Activity: Multiplying and Dividing with Fractions	Groups of 4	25 minutes
	Different groups of 4	20 minutes
	Whole group	20 minutes
Homework and Exit Cards	Whole group	10 minutes

Mathematical Themes

- Students' solutions for sharing situations may result in different additive expressions with fractions. Teachers can help students develop ideas about addition of fractions by challenging them to determine which of their classmates' answers are equivalent and which are incorrect.
- The meaning of multiplication or division may need to be extended when the numbers to be operated on shift from whole numbers to fractions.
- The equivalence of $a \div (b/c)$ and $a \times (c/b)$ can be seen by considering different interpretations of a single diagram.

Connections to the Common Core: Standards for Mathematical Practice

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

MP8 Look for and express regularity in repeated reasoning

Connections to the Common Core: Content Standards

Grade 4: Number and Operations - Fractions 3 and 4
Grade 5: Number and Operations - Fractions 1, 2 and 4

4.NF. 3. *Understand a fraction* ***a/b*** *with* ***a*** *> 1 as a sum of fractions 1/****b****.*
a. Understand addition and subtraction of fractions as joining and separating

parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4. NF. 4. *Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.*

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4).

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5. (In general, n x (a/b) = (n x a)/b.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

5. NF. 1. *Use equivalent fractions as a strategy to add and subtract fractions.*
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

5. NF. 2. *Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

5. NF. 4. *Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.*

a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x q ÷ b. For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general, (a/b) x (c/d) = ac/bd.)

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

An Example of Integrating Content and Practice Standards in MMO

The Standards for Mathematical Practice are intended to be integrated with the Mathematics Content Standards. Students can learn how to engage in these practices only while engaging with content. This facilitator note illustrates how the content and practice standards are integrated in Session Five. Session Three contains a similar note.

In this session, participants dig more deeply into operations with fractions. They focus on representing these operations with pictures, diagrams, story contexts, and symbolic notation. An important part of this work is coordinating fraction notation, expressions, and equations with representations, such as rectangles and number lines, that show the relationships among quantities. The representations bring meaning to the symbols, and illustrate how the same situation might be represented symbolically in more than one way. In the case discussion, participants analyze students' representations and solutions to problems that involve adding fractions. In the math activity, they match multiplication and division problem situations with representations and notation.

The work of this session relates to CCSS content standards across the grades that focus on addition, multiplication, and division with fractions. It includes, for example, the Grade 4 standard that students understand addition of fractions as joining parts referring to the same whole (4.NF.3a) and the Grade 5 standard that students apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction (5.NF.4).

Participants are also working with MP2, Reason abstractly and quantitatively. This practice focuses on moving between problem situations and mathematical abstractions, such as fraction notation. Students might start with a problem situation, and *decontextualize* it by expressing the actions or events that are described in the problem using numbers and symbols. Students might then work with those symbols to determine the solution, but once they have a solution, they interpret their symbols in terms of the context, making sure that their solution makes sense. In Case 22, students work on a problem about sharing brownies. As they solve this problem, they might reason first about the quantities in the problem or they might reason first with the abstract notation. The teacher supports them to move back and forth between the two in order to make sense of both. For example, at one point, Jackson separates the expression, ¼ + ¼ + 1/8, from the context, and tries to solve the problem by adding numerators and denominators. By contextualizing the problem, Jackson realizes that his manipulation of the symbols doesn't make sense, "the numbers I was getting wouldn't match the brownies." In the math activity, participants also work with contexts and expressions, making sense of the multiplication and division expressions in terms of the context and thinking through how the context can be represented by the expressions.

The descriptions of Sessions Three and Five provide examples of how the CCSS content and practice standards are woven together throughout the module. Learning how the CCSS content standards build within and across grades helps participants consider how to bring

coherence and focus to their math instruction. By studying how teachers in the cases explicitly include the math practices in their instruction and by becoming aware of using the practices themselves, participants learn what it means to plan instruction that engages students in these practices embedded in core content. Facilitators should use these examples as a guideline for planning to integrate content and practice standards in remaining sessions of MMO.

Major Focus: *Apply and extend understanding of whole numbers to fractions.*

In Session Five, participants continue to engage with an overarching idea in the Common Core: As the number system is extended from whole numbers to fractions, ideas that were consolidated for whole numbers are revisited to see what stays the same and what needs to be modified to incorporate these new kinds of numbers. In this session, participants will examine multiplication and division with fractions.

Notes to the Facilitator regarding the Standards for Mathematical Practice

MP2 Reason abstractly and quantitatively. *Mathematically proficient students at the elementary grades make sense of quantities and their relationships in problem situations. They can contextualize quantities and operations by using images or stories. They interpret symbols as having meaning, not just as directions to carry out a procedure. Even as they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects.*

In case 22 line 159, when Jackson explains his reasoning, he says he changed his numerical answer because "…the numbers I was getting wouldn't match the brownies." Keeping track of the quantities in the context provides grounding for Jackson to correct his error.

In the math activity, participants use their reasoning within a context to generate both a multiplication and a division expression for the same situation. The context helps them recognize the equivalence of the different expressions. The passage in Maxine's Journal, *Whole-group discussion of the math*, offers an example of this discussion. While Maxine does not cite MP2, you can make this link explicit.

The video clip, Joey's Run, provides examples of the kinds of questions teachers can ask to help students make connections among a context, a representation, and an arithmetic equation.

MP3 Construct viable arguments and critique the reasoning of others. *Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete*

referents such as objects, drawings, diagrams, and actions…. Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. They can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.

In the video clip, Joey's Run, students critique a specific representation and work to understand the reasoning of their classmates. As participants discuss this video, make this link to MP3 explicit.

MP8 Look for and express regularity in repeated reasoning. *Mathematically proficient students at the elementary grades look for regularities as they solve multiple related problems, then identify and describe these regularities.*

Noticing and articulating regularities across examples forms the basis for noticing general principles which can be developed into algorithms that apply to a wide variety of examples. As participants work on Math Activity: Multiplying and Dividing with Fractions, encourage them to articulate general principles such as " A number divided by ½ produces the same answer as that number multiplied by 2." Moving from specific examples to a generalization that applies to all numbers is an example of MP8.

MMO Session Five Agenda Changes linked to Common Core

There are six modifications to the agenda in Session Five.

1. At the beginning of the session, select and share some of the exit card comments from the previous session. If participants wrote about a MP that has yet to be addressed in whole group, be sure to share that comment.

2. Distribute the session overview.

3. A video clip named Joey's Run has been added to this session.

Viewing the DVD: Joey's Run (*Whole group)* (30 minutes)

Before showing the video clip, distribute the handout Joey's Run. Have participants talk to a partner to determine how this representation shows both the elements of the question and the answer. Provide 5 minutes for this conversation.

Let participants know this is a fourth grade class; in previous class sessions all students worked on this problem and drew diagrams to explain their answers. In this clip, the teacher works to have her students understand a particular diagram solution. The clip runs for 7 minutes and 30 seconds. Tell participants to take notes of the teacher's questions and the mathematics highlighted. Suggest they keep MP2 and MP3 in mind as they watch.

In the discussion after the video, first ask participants to explain the mathematics and then turn to the teacher's questions. Comments should include how the teacher's questions help students to make explicit the connections between the context and the mathematical abstraction, especially as they keep track of the units. Make the connection to MP2 if this is not offered by participants. Then discuss the roles of teacher and students in engaging in MP3, Construct viable arguments and critique the reasoning of others.

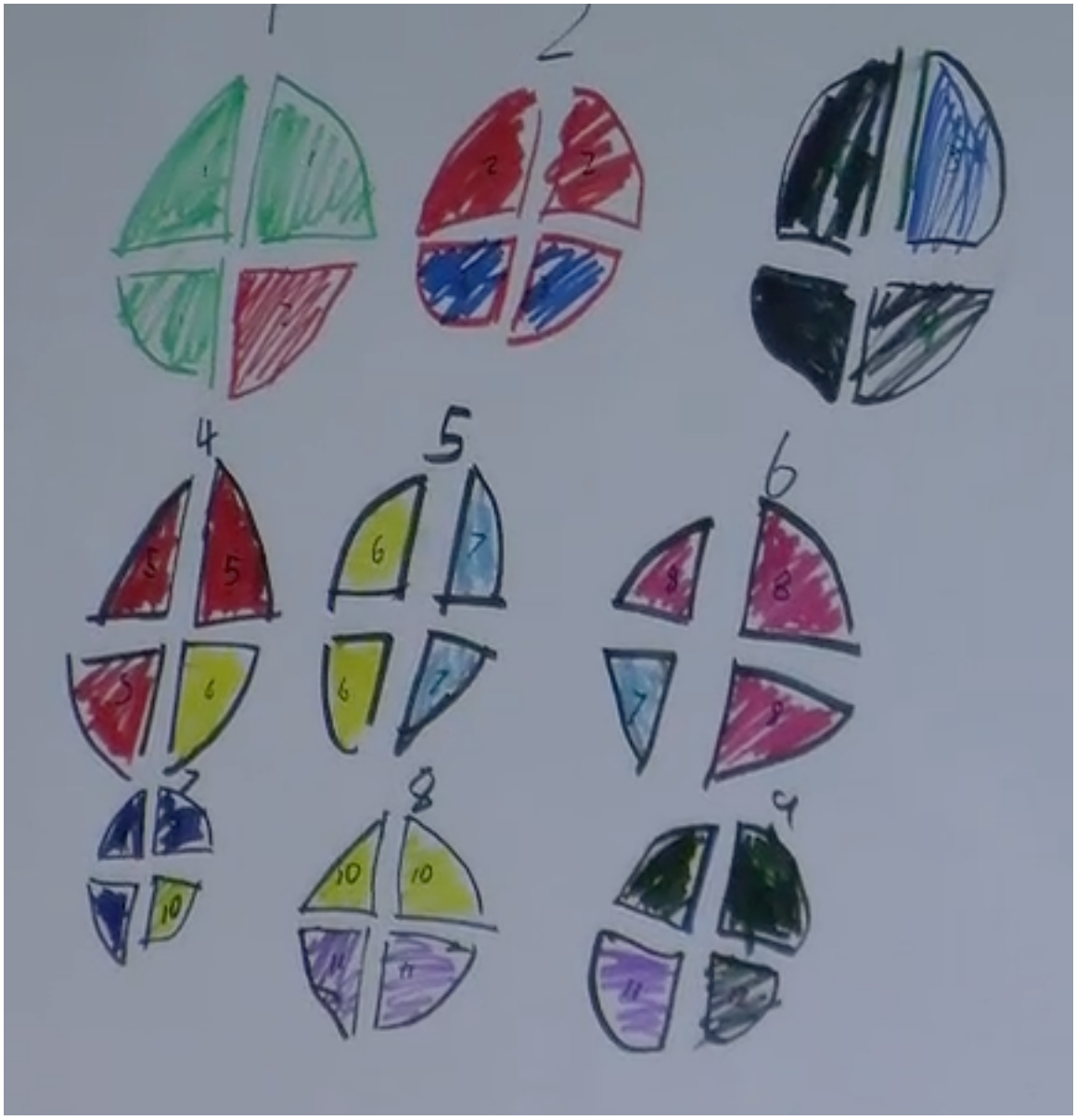
4. During discussions of the math activity, ask questions to help participants connect the elements of their representation and story context to the multiplication and division expressions. At the end of this discussion, provide time for participants to talk about how this work relates to MP2.

5. The DVD clip, Is it multiplication or division, is optional.

6. The Sixth Homework has been revised to include a reference to the mathematical practices. See below for a revised copy of the Sixth Homework.

Joey's Run

Joey ran ¾ mile each day for 12 days. How many miles did he run?



SESSION 5

Sixth Homework

Reading assignment: Casebook chapter 6

In the casebook, read chapter 6, "Taking Portions of Portions, or Multiplying Fractions," including the introductory text and cases 25–27. Consider the questions posed in the introduction as you read the cases.

Writing assignment: Writing about Student Thinking

Pose a mathematics task to your students related to the meaning of fractions (if you teacher younger grades) or operations with fractions (if you teach grade 3 or above). You might pose a question taken directly from one of the cases or the math activities from the seminar.

After the session, think about what happened. What did you expect? Were you surprised? What did you learn? As you listened to the class session, did you notice your students engaged in any of the mathematical practices?

Write up your question, how your students responded, and what you make of their responses (your expectations, your surprises, and what you learned). Include specific examples of student work or dialogue. Include comments on one of the mathematical practices you noticed. Reporting in detail about the work of a few students is very helpful. In particular, it is useful to analyze the work of students whose work might be confusing.

At our next session, you will have the opportunity to share this writing with colleagues in the seminar. Please bring three copies of your writing to share and to turn in.