Session Four Overview

Making Meaning for Operations: Session 4

Agenda

Sharing Exit Card Comments	Whole group	5 minutes
Sharing Student Work: The Brownie Problem	Small groups Whole group	20 minutes 15 minutes
Chapter 4 Case Discussion	Small groups Whole group	30 minutes 25 minutes
Break		15 minutes
Math Activity: Adding Fractional Amounts	Small groups Whole group	30 minutes 30 minutes
Homework and Exit Cards	Whole group	10 minutes

Mathematical Themes

- The numerator and denominator, taken together, determine the value of a fraction.
- Multiplying the numerator and denominator by a constant yields an equivalent fraction.
- Changing the unit results in different fractional names for the same quantity.
- To determine which of two fractions is larger, one may find common denominators and compare numerators, find common numerators and compare denominators, or compare the two fractions to a third number.
- In addition, the addends and the sum refer to the same unit.

Connections to the Common Core: Standards for Mathematical Practice

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

MP4 Model with mathematics.

Connections to the Common Core: Content Standards

Grade 2: Geometry 3
Grade 3: Number and Operations - Fractions 3
Grade 4: Number and Operations - Fractions 1, 2, and 3

2. G 3. *Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.*

3. NF.3 *Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.*

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

4. NF.*1. Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.*

4. NF.2. *Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.*

4. NF. 3. *Understand a fraction a/b with a > 1 as a sum of fractions 1/b.*

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Facilitator Note: *Apply and extend understanding of whole numbers to fractions.*

In Session Four, participants continue to engage with an overarching idea in the Common Core: As the number system is extended from whole numbers to fractions, ideas that were consolidated for whole numbers are revisited to see what stays the same and what needs to be modified to incorporate these new kinds of numbers. In this session, participants will examine how fractions are ordered and how they are added.

Notes to the Facilitator regarding the Standards for Mathematical Practice

MP2 Reason abstractly and quantitatively. *Mathematically proficient students at the elementary grades make sense of quantities and their relationships in problem situations. They can contextualize quantities and operations by using images or stories. They interpret symbols as having meaning, not just as directions to carry out a procedure. Even as they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects.*

In case 18, the teacher suggests students consider a context of cakes as a tool for reasoning about comparing fractions. The students represent the cakes using diagrams which illustrate the relationships among the quantities and interpret the diagrams to represent the size of the quantities. As participants discuss this case; point out the links to MP2.

In the math activity, as participants work to resolve what contexts match $1/5 + 2/5 = 3/5$, ask them to reflect on and analyze their own reasoning in light of MP2. How do the contexts offer new insight into the mathematics? The passage in Maxine's Journal, *Math activity: Adding fractional amounts*, provides additional examples.

MP3 Construct viable arguments and critique the reasoning of others. *Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions…. Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. They can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.*

Throughout the cases in chapter 4, students offer arguments to explain how they know one fraction is greater than or equal to another. In case 18, in particular, many students work to understand and explain Bibiana's diagram. During the case discussion, invite participants to analyze the math discussions and identify examples of students enacting MP3.

MP4 Model with mathematics. *When given a problem in a contextual situation, mathematically proficient students at the elementary grades can identify the mathematical elements of a situation and create a mathematical model that shows those mathematical elements and relationships among them. The mathematical model might be represented in one or more of the following ways: numbers and symbols, geometric figures, pictures or physical objects used to abstract the mathematical elements of the situation, or a mathematical diagram such as a number line, a table, or a graph, or students might use more than one of these to help them interpret the situation.*

The discussion of student work relating to the brownie problem is likely to illustrate students engaged in MP4. If participants notice how their students started with a context, sharing brownies, and then drew out the mathematics by generating expressions and equations to represent the relationships involved, point out that this is an example of MP4. Consult the facilitator note *MP4, Highlighting a common misconception* for additional information. The passage, *Sharing student work: The brownie problem*, in Maxine's Journal offers an example of this discussion. While Maxine does not cite MP4, you should make the connections explicit.

MMO Session Four Agenda Changes linked to Common Core

There are nine modifications to the agenda in Session Four.

1. At the beginning of the session, select and share some of the exit card comments from the previous session. Be sure to include comments about the standards for mathematical practice. If participants wrote about a MP that has yet to be addressed in whole group, be sure to share that comment.

2. Distribute the session overview.

3. Before the small groups start to share their student thinking assignment, remind them to include discussion of the standard(s)for mathematical practice they noted in their students' thinking.

4. A reflection has been added to the focus questions to support discussion of MP2. You might ask, "What are the students doing that illustrate this practice? What are the teachers doing to support this kind of discussion?"

5. The Focus Questions for Chapter 4 has been rewritten to align line numbers in the casebooks with the questions. Some questions previously published have been deleted. See below for a copy of the revised Focus Questions: Chapter 4.

6. Delete the video clip Adding Fractions and add ten minutes to the *Math Activity: Adding fractional amounts*.

7. During the small group and whole group discussion of the math activity, ask questions to help participants connect the elements of their representation and story context to the addition of fraction expressions. At the end of this discussion provide time for participants to talk about how this work relates to MP2.

8. Delete the *Examining Curriculum Choices* assignment. The Fifth Homework has been modified to include a reflection on the Mathematical Practice Standards. See below for a copy of the revised Fifth Homework.

9. Exit Questions for Session 4:

    - What ideas about fractions are clear to you now? What is still confusing?
    - What was the session like for you as a learner?

SESSION 4

Focus Questions: Chapter 4

1. Consider Faith's case 17.

   (a) In the beginning of the case, Harry and Annie use diagrams to explain why 2/3 is greater than 1/2. What ideas about fractions are present in their explanations? What ideas about fractions are left unstated in their explanations?

   (b) Explain the method for comparing fractions used by each of these students: Chuck (lines 21–35), Gary (lines 35-45), Ami (lines 55 -62), and Jesse (lines 75-86). What ideas about fractions does each method depend on?

2. Consider Nelly's case 18.

   (a) Explain the ideas about fractions that you can see in the work of these students: Eli, Rebecca, Matt, and Bibiana.

   (b) Trace the class discussion after Bibiana's poster is shared. Consider the ideas of Matt, Max, and Jillian. What does each student add to the discussion? What ideas does each student figure out? What is still missing?

   (c) What is Kalina's conjecture toward the end of the case (line 195)? Find a fraction that disproves her conjecture.

3. In Malik's case 19 and Zura's case 20, the students work to explain how they know two fractions are equivalent. Use the diagrams in Malik's case to explain why multiplying the numerator and denominator by the same amount produces an equivalent fraction.

Reflections: Look over your work for questions #1-3.
What examples do they offer of MP2, MP3? Be specific and offer an example for each.
Do they illustrate any other of the mathematical practices?

SESSION 4

Fifth Homework

Reading assignment: Casebook chapter 5

In the casebook, read chapter 5, "Combining Shares, or Adding Fractions," including the introductory text and cases 22–24. Use the questions in the introduction to the chapter to guide your reading.

One of the diagrams in Chapter 5 of the casebook was published incorrectly. See below for a corrected version of figure 5.3 Alejandro's work from Case 22 on the brownie-sharing problem. The next page is a corrected version of that diagram.

Writing assignment 1: Thinking about mathematics

This assignment is about the math *you* are learning, not about the math learning of your students. Reflect on the mathematics you have been thinking about in this seminar. Choose one topic to write about. Explain how you thought about this originally, what makes sense to you now, and what aspect of the idea are you still working on. This is an opportunity to share your thinking and questions with the seminar facilitator(s), who will respond to your writing.

Writing assignment 2: Thinking about the Standards for Mathematical Practice

Reread the elaborations of the Standards for Mathematical Practice from the first session of the seminar. Consider how your thinking about these standards has changed over the course of this seminar so far. Choose three specific standards to write about.

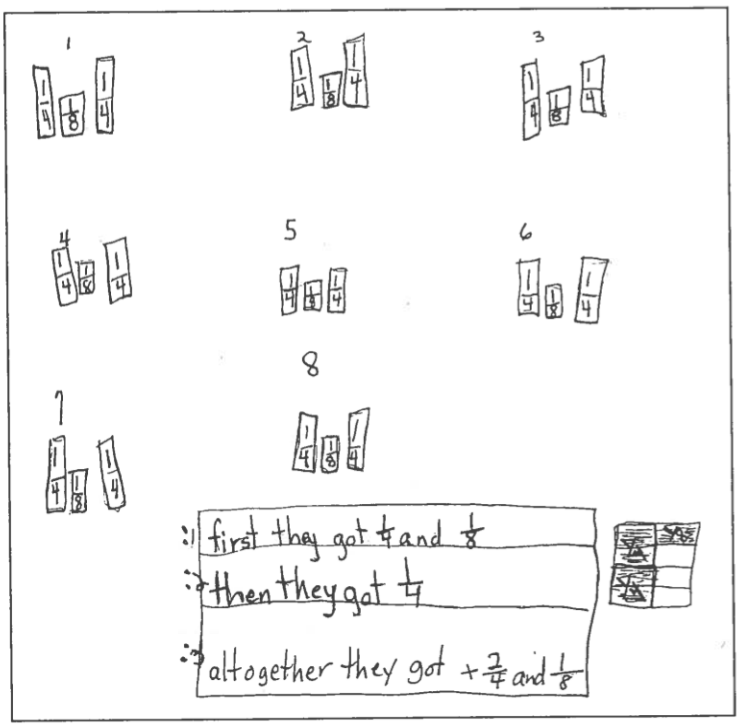

Figure 23. Alejandro's work on the brownie-sharing problem.