Session Two Overview

Making Meaning for Operations: Session 2

Agenda

Sharing Exit Card Comments	Whole group	5 minutes
Sharing Student Thinking	Pairs	20 minutes
Discussion: Norms for learning	Whole group	20 minutes
DVD for Session 2	Whole group	10 minutes
Chapter 2 case discussion	Small groups Whole group	25 minutes 25 minutes
Break		15 minutes
Math activity: Story problems for division	Small groups Whole group Small groups	25 minutes 25 minutes 5 minutes
Homework and exit cards	Whole group	5 minutes

Mathematical Themes

- When working with multiplicative situations, students frequently find it challenging to coordinate the different units: the number of items in a group and the number of groups.
- The variety of students' methods for solving story problems involving multiplication and division illustrates relationships among operations.
- Different kinds of situations can be represented by the same division expression.

Connections to the Common Core: Standards for Mathematical Practice

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

Connections to the Common Core: Content Standards

Grade 2: Operations and Algebraic Thinking 4
Grade 3: Operations and Algebraic Thinking 1, 2, 3,4, 5, and 6

2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

3.OA 1. *Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 x 7.*

3.OA.2. *Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA3. *Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.*

3.OA.4. *Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = □ ÷ 3, 6 x 6 = ?*

3.OA.5. *Apply properties of operations as strategies to multiply and divide. Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56. (Distributive property.)*

3.OA.6. *Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Notes to the Facilitator regarding the Standards for Mathematical Practice

MP2 Reason abstractly and quantitatively. *Mathematically proficient students at the elementary grades make sense of quantities and their relationships in problem situations. They can contextualize quantities and operations by using images or stories. They interpret symbols as having meaning, not just as directions to carry out a procedure. Even as they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent.*

The cases in this chapter provide examples of students actively connecting their number work to the story contexts to develop a deeper understanding of the mathematics. The DVD clip of the third grade student, Ebony, working with the problem how many legs on 4 elephants illustrates how the story context can support a student making sense of the math sentence that represents it. Use questions like, "How does the context support the child's thinking?" "How does this illustrate MP2?" to help participants see these connections.

In the Math Activity: Problems for Division, participants are given an abstract expression, 32 ÷ 5, and create contexts for it, concluding that the contextual situation requires interpretation of the mathematical solution. During this work, participants will be engaged in MP2. Conclude the math discussion with a question such as, "How has this illuminated MP2?" or "What questions does it bring up for you about MP2?" For an example of a seminar discussion that connects contexts to the interpretation of symbols, review the passage of Maxine's Journal titled Math activity: Story problems for division. While Maxine does not cite MP2, you should highlight the links between participants' work and MP2.

MP3 Construct viable arguments and critique the reasoning of others. *Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions…. Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. They can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.*

Session 2 includes a discussion about behavioral norms for the seminar group. Use this opportunity to discuss what it means to construct and critique mathematical arguments. "Critique" does not mean criticize. Rather, it means work to understand another's reasoning. When you don't follow, you pose questions. At times, this challenges one to reconsider or revise one's own argument. Later in the session, toward the end of the math activity, ask participants to identify when they were critiquing one another's arguments, and refer to MP3.

MMO Session Two Agenda Changes linked to Common Core

There are seven modifications to the Agenda in Session Two.

1. Use a few minutes at the beginning of the session to share exit card comments. Be sure to include comments participants wrote about the standards for mathematical practice.

2. Distribute the session overview.

3. Before the small groups start to share their student thinking assignment, remind the group to include discussion of the Standards for Mathematical Practice they noted in their students' thinking.

4. During the discussion regarding Norms for Learning, point out how the norms they generated are related to MP3. Clarify the difference between critiquing and criticizing.

5. A reflection has been added to the Focus Questions to support discussion of MP2. The Focus Questions have also been rewritten to align line numbers in the casebooks with the questions. See below for a copy of the revised Focus Questions: Chapter 2.

6. Exit Questions for Session Two:

    - What was important about the math discussed in this session?
    - How is the seminar working for you as a learner?
    - MP2 and MP3 have been highlighted in this session. Explain something you figured out or are wondering about regarding one of these standards.

7. The writing assignment, Story Problems for $5 \div 8$, has been rewritten to include a reference to MP2. See below for a copy of the revised Third Homework.

SESSION 2

Focus Questions: Chapter 2

1. In Bella's case 8, children are working to answer questions that we would consider multiplication. Consider the representations offered by Jason, Rashad, Carlita, Kenya, and Flora. How does each illustrate multiplication? How are they the same and how are they different?

2. Also in Bella's case, consider the thinking of Junior in lines 78 to 92. What ideas about multiplication are he is grappling with? What does his confusion illuminate about the nature of multiplication?

3. In Janine's case 10, we see children working on a multiplication problem, making mistakes and sorting out their confusions. Where did these children get confused? How did they sort it out? How are the ideas in this fourth-grade class similar to Junior's confusion in Bella's kindergarten case?

4. Consider Melinda's case. Explain the thinking of Su-Yin, and also that of Derrick and William. What ideas about multiplication and division are highlighted by the thinking of these children?

Reflection: Look over your work for questions #1-4. Locate one or two examples that illustrate how the work of these students is related to MP2. Be specific as you explain how your example illustrates MP2.

5. In Nisha's case 9, we see second graders writing arithmetic expressions to represent the ways they determined the total number of tiles in an arrangement of 15 tiles. What ideas about multiplication are they developing? What connections are there between this work in second grade and future work on multiplication?

6. In Georgia's case 11, we see students using addition, subtraction, and multiplication to solve problems that most of us would consider division problems.

   (a) In particular, consider Vanessa's work on the first two problems (lines 315 to 330). Vanessa subtracts to solve one problem and adds to solve the other. Why do you think she uses different operations for these two problems?

   (b) Consider the work of Cory (lines 335 to 350) and Matthew (lines 360–380). What does each student's approach indicate about his thinking about division?

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Third Homework

Reading assignment: Casebook chapter 3

In the casebook, read chapter 3, "When Dividing Doesn't Come Out Evenly," including the introductory text and cases 13 to 16. Consider the questions posed in the chapter introduction as you read.

Writing assignment: Story problems for 5 ÷ 8

You may have worked on this problem briefly in the seminar. This assignment provides the opportunity for you to revisit your thinking by writing your own answers. Your writing can also include questions you have about any parts of the problem that are still confusing or unclear. Seminar facilitators will read what you have written and respond to your ideas and questions.

1. Write story problems for 5 ÷ 8 so that the question you pose would be answered by each of the following.

   (a) 5/8 (b) 0.625 (c) 0 or 1 (d) 0 (e) 1

   One way to approach this task is to examine the stories your group wrote for 32 ÷ 5 and determine how to modify them to match 5 ÷ 8.

2. For each problem, explain what it is in the story context that makes the answer appropriate for the situation.

3. Describe how your math thinking offers an example of MP2.