

Session Five Overview

Building a System of Tens: Session 5

Agenda

Sharing Exit Card Comments	Whole group	5 minutes
Sharing Student Thinking Assignments	Groups of 3	35 minutes
Chapter 5 Case Discussion	Small groups	25 minutes
	Whole group	15 minutes
Break		15 minutes
DVD for Session Five	Whole group	40 minutes
Math Activity: Procedures for Division	Small groups	30 minutes
	Whole group	15 minutes
Homework and Exit Cards	Whole group	10 minutes

Mathematical Themes

- Computational strategies for multi-digit division rely on the place value system and properties of the operations.
- Examining the reasoning that underlies division procedures, including the standard algorithm, provides a context for students to deepen their understanding of the place value system and of the operations themselves.
- Each visual, physical, or verbal representation of division highlights some aspects of the mathematics and obscures others.

Connections to the Common Core: Standards for Mathematical Practice

MP3 Construct viable arguments and critique the reasoning of others.

MP7 Look for and make use of structure.

MP8 Look for and express regularity in repeated reasoning.

Connections to the Common Core: Content Standards

Grade 4: Number and Operations in Base Ten 6

Grade 5: Number and Operations in Base Ten 6

Grade 6: The Number Systems 2

4.NBT.6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.6: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

6.NS.2: Fluently divide multi-digit numbers using the standard algorithm.

Note: The Common Core and the US Traditional Algorithms

In the Session Five case discussion and video clip analysis, participants examine student strategies for division and explain how these strategies call upon the meaning and properties of multiplication and division. In the math activity, participants analyze the traditional division algorithm. The Common Core approach to algorithms is similar: Formal work with the algorithms comes years after students begin to explore computation with other methods. In grades 6, they are expected to learn the standard division algorithm.

Notes to the Facilitator regarding the Standards for Mathematical Practice

MP3 Construct viable arguments and critique the reasoning of others. *Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions.... Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. They can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.*

As participants analyze each other's approaches to division, they engage in MP3. Call attention to comments they make explaining each other's approaches and how their own thinking has changed as a result of their discussions.

MP7 Look for and make use of structure. *Mathematically proficient students at the elementary grades use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference), and attributes of shapes to solve problems*

In the case and DVD clip discussions, participants examine how the base-ten decomposition of numbers can be useful in performing division. Other structures applied in the division strategies include the distributive property and the relationship between multiplication and division. As participants work on the math activity and case discussion, point out examples of mathematical structure or ask questions to highlight such use of structure.

MP8 Look for and express regularity in repeated reasoning. *Mathematically proficient students at the elementary grades look for regularities as they solve multiple related problems, then identify and describe these regularities.*

In the Session Five case discussion, participants examine how two students (Jen and April) each solve a particular division problem. They are encouraged to try other division problems using these approaches and finally to generalize a strategy that will always work.

This process is an example of expressing regularity in repeated reasoning. Refer to Maxine's Journal line numbers 73 to 141 for an example of this kind of discussion. While Maxine does not explicitly cite MP8, you should make the connection explicit.

BST Session Five Agenda Changes linked to Common Core

There are eight modifications to the agenda in Session Five.

1. Use a few minutes at the beginning of the session to share exit card comments.
2. Distribute the session overview.
3. During the Chapter 5 case discussion, add this bullet to the three on the list to be posted as the overarching questions:
 - How is MP7 enacted in these cases?
4. A new focus question (Reflection) for the small-group case discussion has been added. See below for a copy of the new Focus Questions.
5. When viewing the DVD clips, ask participants to discuss which student approaches call upon a "groups of 13" image of division and which call upon a "make 13 equal groups" image. Analyzing the approaches with these two meanings of division will be important as they move to the math activity, Procedures for Division.
6. The math activity: Procedures for Division has been revised. Remind participants that it is not expected that they complete all the questions. If most of the groups have worked on questions 1, 2, and 3, focus the whole group discussion on #3 by having participants share and explain their different approaches. See below for a copy of the new math activity.
7. The Sixth Homework has been redesigned. See below for a copy of the new assignment.
8. The Examining Curriculum Activities Choices and Examining Curriculum Activities Assignment have been deleted.

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Focus Questions: Chapter 5

1. In Eleanor's case 20, explain Jen's approach as she divides 121 by 3. Try a few other division problems using Jen's strategy. Write a verbal description of her strategy. Will this strategy always work? Explain why or why not. Use story contexts, diagrams, and/or cube representations as part of your explanation.
2. In Janine's case 21, April decides to divide the 143 jellybeans among 8 people by seeing first how many 8s there are in 100 and then how many 8s there are in 43. She goes on to divide up the remaining 7 jellybeans. Try another division problem using April's strategy. Write a verbal description of her strategy. Will this strategy always work? Explain why or why not. Use story contexts, diagrams, and/or cube representations as part of your explanation.
3. In Susie's case 19, the students are talking about what happens to a number when you divide by 10, and also what happens when you multiply by 10. In lines 75 to 84, James and Alli offer their ideas about multiplying by 10. Explain what they are thinking. In lines 85 to 97, Sadie, Alli, and James refer to Sadie's arithmetic (lines 60 to 67) to talk about what happens when you divide a number by 10. Explain what they are thinking.

Reflection: Explain the connections between the students' activity in these cases and MP7 Look for and make use of structure, MP8 Look for and express regularity in repeated reasoning, and MP3 Construct viable arguments and critique the reasoning of others.

Math Activity: Procedures for Division

(Note: It is more important to think through each problem deeply than to finish all the problems by the end of the session.)

1. Examine Lexie's approach to $369 \div 3$. What image of division connects to her strategy? What connections do you notice between the method of recording her work in Figure 5.2 and the traditionally taught division algorithm? Try Lexie's strategy on $78 \div 3$. What do you notice?
2. Try Lexie's strategy on $78 \div 3$ and $176 \div 13$. What do you notice?
3. In session 2, we examined four procedures for computing multi-digit addition problems and five for multi-digit subtraction problems. Consider $176 \div 13$ or a similar division problem. Show as many procedures for calculating this problem as you can. Explain how each one works.
4. In this seminar we have examined and analyzed various procedures for computations for addition, subtraction, and multiplication. Consider problems such as $79 \div 3$ and $176 \div 13$. Solve the problems using the traditionally taught algorithm - the one modeled by Kevin on the DVD segment - and then model the steps of the procedure with the base-ten blocks and arithmetic expressions. What are the connections between place value ideas and this division procedure?

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Sixth Homework

Reading assignment: Casebook chapter 6

In the casebook, read chapter 6, “Place Value Representation of Numbers Less Than 1,” including the introductory text and cases 23–27. Use the questions posed in the introduction to guide your reading.

Writing assignment 1: Thinking about mathematics

This assignment is about the math *you* are learning in the seminar, not about the math learning of your students. Reflect on the mathematics you have been thinking about in this seminar. Choose one topic to write about. Explain how you thought about this originally, what makes sense to you now, and what aspect of the idea are you still working on.

Writing assignment 2: Thinking about the Standards for Mathematical Practice

Reread the elaborations of the Mathematical Practice Standards from the first session of the seminar. Consider how has your thinking about these Standards has changed over the course of this seminar so far. Choose three specific standards to write about.