Session Four Overview

BST Session Four

Agenda

Sharing Exit Card Comments	Whole group	5 minutes
Math activity: Multidigit Multiplication	Whole group Small groups Whole group	10 minutes 35 minutes 30 minutes
Break		15 minutes
DVD for Session Four	Whole group	25 minutes
Chapter 4 Case Discussion	Small groups Whole group	30 minutes 25 minutes
Homework and Exit Cards	Whole group	5 minutes

Mathematical Themes

- Multiplication problems can be represented in different ways; each representation highlights some aspect of multi-digit computation and obscures others.

- Procedures for calculating a multi-digit multiplication problem rely on both properties of operations and the base ten structure of number.

- As students work to understand the reasoning behind both alternative and the traditional computational procedures, they deepen their knowledge of multiplication.

Connections to the Common Core: Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

MP7 Look for and make use of structure.

Connections to the Common Core: Content Standards

Grade 4: Number and Operations in Base Ten 5
Grade 5: Number and Operations in Base Ten 2 and 5

4.NBT.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

5.NBT.5: Fluently multiply multi-digit whole numbers using the standard algorithm.

Note: The Common Core and the US Traditional Algorithms

In the math activity for Session Four, building on the work of how multi-digit numbers can be decomposed, participants explore strategies for multiplying multi-digit numbers and examine the connections between these approaches and the traditional US algorithm. The Common Core approach to algorithms is similar: In grades 2, 3, and 4, students become fluent in a variety of multiplication strategies, and by the end of grade 5 they are expected to know the standard algorithms for multiplication.

Notes to the Facilitator Regarding the Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them. *Mathematically proficient students at the elementary grades explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway.*

In one of the teacher interviews on the DVD, Nancy Horowitz describes how Nicholas was stuck on a multiplication problem. She asked what multiplication facts he already knew that might help him, and this was all he needed to continue to work. This question, which helped Nicholas find a way into the problem, is related to one that mathematicians ask themselves regularly: If I don't know how to solve this problem, is there a related problem, perhaps a simpler problem, that I do know how to solve? Often, work on the related problem provides insight that leads back to the original problem. This is an aspect of MP1.

MP2 Reason abstractly and quantitatively. *Mathematically proficient students at the elementary grades make sense of quantities and their relationships in problem situations. They can contextualize quantities and operations by using images or stories.*

By creating story contexts and representations for 16 x 18, participants develop deeper meaning for the abstract symbols. As you interact with small groups during the math activity, ask how their diagrams or representations fit the various elements of the story context, and help them make connections between the components of the story, the representation, and their mathematical symbols. For example, if participants used the multiplication algorithm to find the product of 16 x 18, ask them to identify where each line in their calculation appears in their diagram and what it means in terms of their story. At the end of the math discussion, invite comments MP2 by asking participants to talk about how the story context and representations drew them to new insights about multidigit multiplication. You can also refer to Maxine's Journal line numbers 20 to 104 for an example of this kind of discussion. While Maxine does not explicitly cite MP2, you should make the connection explicit.

MP3 Construct viable arguments and critique the reasoning of others. *Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions…. Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. They can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.*

After viewing the DVD clip of fifth graders explaining their methods, ask participants to comment on how they see MP3 enacted. In particular, the fifth grade class was asked to critique Thomas's incorrect strategy in order to explore more deeply *why* it doesn't work (when, on the surface, it seems like a reasonable approach) and to understand what *does* work if they begin with Thomas's initial steps. What aspects of a classroom culture should be put in place in order to make that kind of activity productive for Thomas and for his classmates?

MP7 Look for and make use of structure. M*athematically proficient students at the elementary grades use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference), and attributes of shapes to solve problems.*

As participants work on the math activity, they use various mathematical structures: place value, the meaning of multiplication, and the distributive property of multiplication over addition.

One goal of the math activity is to help participants see how multiplicative structures differ from additive structures. As participants share representations for the multi-digit multiplication problems, ask questions such as, "Why doesn't the strategy used for addition (such as, first add the tens and then add the ones) work for multiplication?"

BST Session Four Agenda Changes linked to Common Core

There are seven refinements to the agenda in Session Four.

1. At the beginning of the session, select and share some of the exit card comments from the previous session. Be sure to include comments about MP8 to offer clarifications.

2. Distribute the session overview.

3. The math activity has been modified. Below is a copy of the revised math activity. A main focus of the math activity is to sort out how multiplication behaves differently from addition.

4. During the small group and whole group discussion of the math activity, ask questions to help participants connect the elements of their representation and story context to the multiplication expressions. Ask participants how making those connections deepen understanding of the abstractions. At the end of this discussion, point out to participants that this is the meaning of MP2.

5. In the discussion of the DVD clip from a fifth grade class, pose a question about the teacher's moves in the context of MP3: "What is it the teacher is doing to support the students in learning to compare and critique each other's approaches? What aspects of a classroom culture should be put into place to make such critiques a productive activity for all students?"

6. Revised exit card questions for Session Four:

    - What new math ideas were brought up for you in this session?
    - What was the session like for you as a learner?
    - What Standards for Mathematical Practice did you engage in in this session?

7. The student thinking assignment has been rewritten to include references to the Standards for Mathematical Practice. See below for the rewritten assignment.

SESSION 4

Math Activity: Multidigit Multiplication

(Note: It is more important to think through each problem deeply than to finish all the problems by the end of the session.)

1. Write a word problem that would be solved by 16 × 18.

2. Create representations of 16 × 18 with diagrams, base ten blocks, or cubes.

3. Frequently students try to solve multiplication problems by using strategies that have worked for them in addition. For each approach (a), (b), and (c)
   - Explain why it might seem to be a good approach.
   - Explain what is incorrect about the approach.
   - Explain how to modify the approach so it leads to a correct answer.

   Use story contexts, diagrams, and other models in your explanations.

   (a) To find 16 + 18, I added the tens (10 + 10 = 20) and I added the ones (6 + 8 = 14), then added those together to get the answer (20 + 14 = 34). Why not solve 16 × 18 by computing (10 × 10) + (6 × 8)?

   (b) To find 36 + 17, I added 4 to the 36 and 3 to the 17 to make the problem easier (40 + 20 = 60). Then I subtracted the extra 4 and the extra 3 to get the final answer (60 – 4 – 3 = 53). Why not solve 36 × 17 the same way? 40 × 20 = 800, and then subtract the extra 4 and the extra 3 to get 800 – 4 – 3?

   (c) To find 16 + 18, I took 2 from the 16 and added it to the 18, making the expression 14 + 20. Why not solve 16 x 18 by solving 14 x 20?

4. Perform these two calculations using the standard algorithm:

   $$\begin{array}{r} 16 \\ \times\ 18 \\ \hline \end{array} \qquad \begin{array}{r} 18 \\ \times\ 16 \\ \hline \end{array}$$

   Compare the results of these computations with the diagrams and story contexts you used for 3a. What mathematical structures does the traditional US algorithm call upon?

5. Explain how you can modify the approach you used in 3(a) to perform this multiplication: (x + 6) (x + 8).

SESSION 4

Fifth Homework

Reading assignment: Casebook chapter 5

In the casebook, read chapter 5, "Division with Multidigit Numbers," including the introductory text and cases 19–22. The questions in the introduction are useful to keep in mind as you read the cases.

Writing assignment: Student thinking

It is likely that reading the cases and working on the mathematics in this seminar have made you curious about how your own students might work with multidigit computations. This assignment asks you to examine the thinking of your students.

Select a mathematics task for your students related to computation of multidigit numbers. (If you work with students for whom this might not be appropriate, focus on the way they understand numbers between 10 and 20.) You might pose a question taken directly from one of the cases or from the math activities from the seminar, such as exploring a subtraction or multiplication problem using mental math, and representations such as number lines or base ten drawings.

After the session, think about what happened. What did you expect? Were you surprised? What did you learn? As you listened to the class session, did you notice your students engaged in any of the Mathematical Practice Standards?

Write up your question, how your students responded, and what you make of their responses (your expectations, your surprises, and what you learned). Include specific examples of student work or dialogue. Include comments on one of the Mathematical Practice Standards you noticed. Reporting in detail about the work of a few students is very helpful. In particular, it is useful to analyze the work of students whose work might be confusing.

At our next session, you will have the opportunity to share this writing with colleagues in the seminar. Please bring three copies of your writing to share and to turn in.