Using Story Situations to Illuminate the Meaning of the Operations

by Virginia Bastable, Mount Holyoke College
vbastabl@mtholyoke.edu

A third grade student was asked to explain why the number sentence 19 + 6 = 20 + 5 is true. In response she offered the following story situation:

> *If I had some candy and I shared with my friend, but then I decided to share more with her, we would still have the same amount even though I'm sharing more with her. If I had 20 pieces and my friend had 5 pieces the sum would be 25. But then if I gave her another one of my pieces so she has 6 we would still have 25 together. So it doesn't matter how we share the candy the total will always be the same. Unless we go get more or we eat some of it.*

The story problem about sharing candy provides an image that illustrates not just that the two sums are the same but *why* they must be the same. The student uses the story context to explain that the equality will hold "unless we go get more or we eat some of it." Students frequently explain such equality by saying, "You take one off the 6 and add it to the 19." The story situation adds an important element to student understanding of how addition works. With a focus on the two friends each with his/her own amount of candy, one way to express the arithmetic that matches the actions in the story would be:

$20 + 5 = (19 + 1) + (6 - 1) = 19 + 6$.

The number sentence that prompted this analysis could have arisen in different ways. It may have been generated by work on a routine familiar to many teachers, Today's Number: "How many different ways can you make 25?" Or it may have been a computational strategy: "To solve 19 + 6, I'll change it to 20 + 5 because that is easier to add." In either case, the story situation highlights an important behavior of addition: when adding two numbers, if you add something to one addend and subtract the same amount from the other addend the sum remains the same.

The story about sharing candy came from a teacher who was enrolled in an online course based on the book, *Connecting Arithmetic to Algebra* (Russell, Shifter, Bastable 2011). In this course, teachers consider generalizations that arise from the study of operations in grades 1 through 6 by examining cases of students engaged in the process of articulating general claims, working to understand those claims, and learning how to prove them. This article will focus on one aspect of the *Connecting Arithmetic to Algebra* work, using story situations to illustrate the behavior of the operations.

While many educators recognize the value of multiple representations and encourage the use of manipulatives, number lines, hundreds charts, or diagrams, considering a story situation as a representation is less common. In this article I will examine this topic through three classroom examples.

Classroom Example One

In the first example, the story about sharing candy provides a structure that the students in the class can use to continue to explore addition. They can articulate the meaning of addition as combining all the candy. They can modify the story by changing the amount of candy each friend has. They can modify the amount of candy one friend gives to the other. These kinds of explorations based on the story situation could eventually lead them to make an argument that no matter how many pieces of candy you begin with and no matter how many you give to your friend, the number of candies remains constant, proving for all positive whole numbers (as long as x is smaller than or equal to either a or b), that $a + b = (a + x) + (b - x)$.

The goal of such work is not for students to generate the symbolic forms, but for students to generate mental images that are associated with and carry the meaning of the oper-

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ation of addition. Frequently, students assume that regularities they notice about one arithmetic situation are true in general and the significance of the operation is lost. For instance, 19 + 6 and 19 – 6 look similar on the page, but what if we move 1 from the 6 and add it to the 19 in the subtraction expression. Is 19 – 6 = 20 – 5? We see it is not true. By investigating if claims that are true for addition will still work with subtraction, students come to see that approaches that work for one operation may not be true for another. They begin to see the operations as distinct from one another, each with its own set of behaviors.

The two friends and candy story becomes a grounding for addition. New stories are needed to generate mental images for subtraction. What kind of stories will capture the action of subtraction? How can those stories be used to examine what happens with changes in subtraction expressions? Working on these ideas explicitly is valuable for students since it allows them to develop rich and complex conceptions for how each of the operations is different from the others.

Classroom Example Two

In the next example, a third grade class is working on the idea that, if you add an amount to one addend, the sum is increased by that amount. One student wrote this story as an illustration.

Mr. B went to the store to get tomatoes and eggs. He got 3 tomatoes and 6 eggs which equals 9 vegetables in all. Then when he got to the checkout, the cashier said he could get a discount instead of 6 eggs, 8 eggs instead. How many vegetables does Mr. B have now?

3 + 6 = 9; 3 + 8 = 11

This third grader has created a story context to show that if you add 2 to an addend, you also add 2 to the sum. She has yet to learn to what category of food eggs belong, but the story situation she devised both matches the original number sentence (3 + 6 = 9) and also captures the idea of changing an addend by 2.

This class could further refine this story by explicitly articulating that the new total is made up of two parts, the original sum plus the 2 additional eggs. In that case, the story would match the arithmetic sentence 3 + (6 + 2) = (3 + 6) + 2. With this version, we see an example of the associative property of addition. The Common Core State Standards document includes the properties of the operations as an important topic, while also specifically stating that it is not necessary for students to use these formal terms at a young age. These examples illustrate how devising story situations that capture the actions behind arithmetic sentences can help students notice how their arithmetic strategies take advantage of these properties.

Classroom Example Three

Consider this final example. A third grade class was working on the idea that addition and subtraction have an inverse relationship. One student wrote the following story.

Megan has 4 dogs. Molly goes on vacation. Molly wants Megan to babysit her dogs. Two days later Molly comes back and gets her dogs back. How many dogs does Megan have now?

In the class discussion that followed, students first stated that the story situation is incomplete because it did not include the number of dogs that Molly owns. In the conversation that followed they began to notice that the story matches the situation without needing that piece of information. We join the class, with the teacher asking why they do not need to know how many dogs Molly has.

Student: Because if Molly picks up her dogs, however many she has, she takes, then whatever number minus that number will give you four. So you just. . . Molly's just taking her dogs home with her, and well, you don't know how many she has.

Teacher: Could we act this out to make it clearer?

Two students stand up to act out the problem while the remaining students direct them. The first student holds up four fingers to represent Megan's four dogs. The second student joins the first wiggling her fingers to indicate "however many" dogs. Then the second student walks away, representing Molly taking her "however many" dogs home.

Students: What's left is Megan and her four dogs.

In this excerpt the class is analyzing how a story situation can be used to illustrate the re-

lationship between addition and subtraction. In the language of these third grade students, the story matches the sentence 4 + "however many" – "however many" = 4. In more formal terms, it might be expressed as $4 + x - x = 4$. The action of the story captures this particular relationship between addition and subtraction. The thinking of the students is within their work in arithmetic, but they are developing concepts that will serve them when they encounter formal algebra.

Conclusion

These three examples offer a glimpse into the value of using story situations to help students uncover, notice, and articulate some of the properties of the operations that underlie their arithmetic work. By representing arithmetic expressions and equations with story situations, students can ground their thinking in mental images that capture the meaning, behavior, and properties of the operations. The story situations become tools for their reasoning, not application problems to solve.

While this article has focused on addition and subtraction, the same approach is productive for multiplication and division as well. I will leave you with two challenges:

1. Create a story situation to explore this question: What happens to the product if you add 1 to one of the factors?
2. Create a story situation to explain why this number sentence is true: 4 x 10 = 8 x 5.

Reference

Russell, S.J., D. Schifter, V. Bastable. 2011. *Connecting Arithmetic to Algebra: Strategies for Building Algebraic Thinking in the Elementary Grades.* Portsmouth, NH: Heinemann.

Writing to Learn in Mathematics

by David L. Fama
Royal Crest Academy, Front Royal, Virginia

Many of the recent changes in the mathematics curriculum and pedagogy focus on increasing student proficiency in writing about mathematics. According to a recent study concerning the retention of information the percentage breakdown is as follows:

10% of what they read
20% of what they hear
30% of what they see
50% of what they both hear and see
70% of what they discover
80% of what they experience
90% of what they teach

The accompanying research project was developed to incorporate the last three retention percentages: to discover, to experience, and to teach. A project of this kind is actually a research essay designed to enhance conceptual understanding.

Any well-designed research project should have the following objectives:

- ✔ to promote the clarification and organization of ideas and concepts
- ✔ to stimulate problem articulation and analysis
- ✔ to enhance pattern discernment and description
- ✔ to develop critical thinking skills
- ✔ to generate creative thinking skills
- ✔ to facilitate interaction between the instructor and the student, since writing is inherently an active and not a passive activity.

The following research project was de-

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